5/26/05 Draft

Timeout for Euclid, Einstein, and Evenson: Wave Geometrology for Relativity and Quantum Theory

William G. Harter
University of Arkansas

Plane-wave-geometry and laser metrology provide a Euclidian logic that connects Einstein's special theory of relativity to Planck's quantum theory. By considering a two thousand year span of ideas in the light and great precision of modern optics, one arrives at a more concise and precise logic that gives better intuition for two of the foundations of modern physics.

Contents

-- The Purest Light and a Resonance Hero – Ken Evenson (1932-2002) --

When travelers punch up their GPS coordinates they owe a debt of gratitude to an under sung hero who, alongside his colleagues and students, often toiled 18 hour days deep inside a laser laboratory lit only by the purest light in the universe.

Ken was an "Indiana Jones" of modern physics. While he may never have been called "Montana Ken," such a name would describe a real life hero from Bozeman, Montana, whose extraordinary accomplishments in many ways surpass the fictional characters in cinematic thrillers like *Raiders of the Lost Arc*.

Indeed, there were some exciting real life moments shared by his wife Vera, one together with Ken in a canoe literally inches from the hundred-foot drop-off of Brazil's largest waterfall. But, such outdoor exploits, of which Ken had many, pale in the light of an in-the-lab brilliance and courage that profoundly enriched the world.

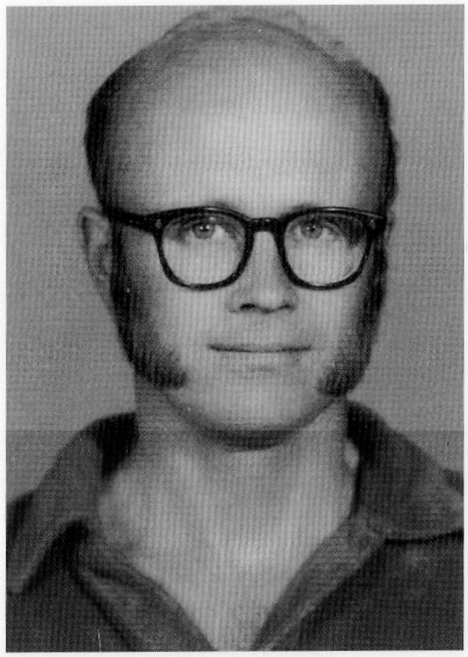
Ken is one of few researchers and perhaps the only physicist to be twice listed in the *Guinness Book of Records*. The listings are not for jungle exploits but for his lab's highest frequency measurement and their speed of light determination that made c many times more precise due to his pioneering work in laser resonance and metrology.

Then the meter-kilogram-second (mks) system of units underwent a redefinition largely because of Ken's efforts. Thereafter, the speed of light c was set to $299{,}792{,}458 ms^{-1}$. The meter was defined in terms of c, instead of the other way around since his time precision had so far trumped that for distance. Without such resonance precision, the Global Positioning System (GPS), the first large-scale wave space-time coordinate system, would not be possible.

Ken's courage and persistence at the Time and Frequency Division of the Boulder Laboratories in the National Bureau of Standards (now the National Institute of Standards and Technology or NIST) are legendary as are his railings against boneheaded administrators who seemed bent on thwarting his best efforts. Undaunted, Ken's lab painstakingly exploited the resonance properties of metal-insulator diodes, and succeeded in literally counting the waves of near-infrared radiation and eventually visible light itself.

Those who knew Ken miss him terribly. But, his indelible legacy resonates today as ultra-precise atomic and molecular wave and pulse quantum optics continue to advance and provide heretofore unimaginable capability. Our quality of life depends on their metrology through the Quality and Finesse of the resonant oscillators that are the heartbeats of our technology.

Before being taken by Lou Gehrig's disease, Ken began ultra-precise laser spectroscopy of unusual molecules such as HO_2, the radical cousin of the more common H_2O. Like Ken, such radical molecules affect us as much or more than better known ones. But also like Ken, they toil in obscurity, illuminated only by the purest light in the universe.



PAULINIA, BRASIL 1976

THE SPEED OF LIGHT IS
299,792,458 METERS PER SECOND!

Kenneth M. Evenson – 1932-2002

"I'm just a wi-ild and cra-aazy guy!" was an opening by humorist Steve Martin who made a career satirizing late 20th century American *Zeitgeist* including tongue-out images of Albert Einstein, our most famous adopted physics icon. Many get comfort from a myth that a few crazy ideas by Einstein turned centuries of physics and geometry on its head. Recent postmodern physicists boast of an "Einstein complex" seeking the next wild idea to revolutionize physics.[1] Meanwhile, outside of physics, postmodern deconstruction movements rage to the verge of dismissing logic itself as just another belief system.[2]

We need to remind ourselves that for Martin and Einstein, craziness was just an act. To be successful, each had to be quite sane. Also, each had the prescience to warn of an era such as ours where a nation and world are going mad. Einstein saw by horrific example that even physics is not immune to a prevailing social commonality or a regressing *Zeitgeist*.

The 2005 AIP Year of Physics and Einstein Centennial provides a time-out to examine *evidence* and *logic* of Einstein's work and link it to ideas occurring before and since 1905 going way back to Euclid. Here we sketch a clearer perspective designed to show how evidence and logic connect modern and earlier physics and not just simplistic tradition, faith or fancy.

Since 1905, experimental evidence has increased at least a million-fold in both scope and precision. Some of this receives spectacular coverage with Hubble images and big-bang effects, but a lot of the most precise results are hidden from public view in darkened labs for laser optics, spectroscopy, and BEC. This article tries to show how more precise metrology can lead to more precise logic and thereby reveal a surprisingly tight chain of ideas thousands of years old.

Back to the drawing board

Students and laymen deserve nothing less if societal benefits are to outlast the AIP Year and aid NSF education initiatives to help faltering postmodern US students. To this end, the following shows a new-but-old hands-on and back-to-basic-geometry approach to classical and modern physics wherein students take notes on graph paper using a drawing board, square rule, and compass, and in so doing, rediscover the oldest Weapons of Math Instruction (WMI).[3]

Logical development, including Einstein's relativity as shown below, is taught first by analytic-geometric construction that then motivates derivation by algebra or calculus. (Then one may use a "techno-crutch" (calculator) to verify geometric "experiment" or improve precision.)

We recall that Newton's *Principia* puts geometry first, too.[4] However, *Principia* figures can be pedagogical nightmares of tangled arcs. To save paper, Newton drew geometric steps on top of one another so their logic did not catch on. (If his calculus had been so juxtaposed it might not have caught on either.) But, technology makes geometry easier using graph paper, cool pens or (for the lazy) computer graphic CAD systems so each logical step is a separate figure or layer.

This reopens a delightful and insightful mode of human thought, the logic of Euclidian geometry.

Basic baseline geometry

To show the power and insight gained by geometric logic we demonstrate a construction of the fundamentals of special relativity. It turns out that in a few steps with a ruler and compass one may derive qualitative and quantitative fundamentals of *both* special relativity *and* quantum mechanics and clarify the logic of both subjects without using lightning, smoke or mirrors.

That, you might have said a year ago, is about as likely as having the Boston Red Sox win the World Series! But, win they did and in their honor we begin our construction around a baseball diamond in Fig. 1a. Let first and third base represent, respectively, left-to-right *(L-to-R)* and right-to-left *(R-to-L)* laser beams from *600 THz* lasers that each project a beautiful emerald green beam of light whose wavelength is one half of a millionth of a meter. ($\lambda = 0.5\mu m$)

Fig. 1a-b plots frequency $\nu=\omega/2\pi$ or "wiggle" rate ω along the vertical axis, that is, the number of waves put out by each laser per unit of *time*, and the horizontal κ-axis plots number of waves per unit of *space*, that is, $(\kappa=1/\lambda)$-"kinks" per meter. We coin a term *per-spacetime* for plots of "wiggle" *versus* "kinkiness" such as ν *vs.* κ or ω *vs. ck*. (κ relates to k by $\kappa=k/2\pi$.)

In Fig. 1 ν is *600 teraHertz* and ω is *1200π tera-radians per second*. In the 1970's Ken Evenson[5] and Jan Hall were first to count such high frequency to a precision of 11 or 12-figures. Their feats were noted in the *Guinness Book of Records* along with Evenson's speed of light[6] measurement that led to a 1980 redefinition of the meter. This made these modern metrologists, like Red Sox fans, heroes of patience. Ever since Galileo's swinging chapel lamps, oscillator physics has aided metrology. In recent decades, oscillator Quality[7] has improved enormously.

Wavelength λ and wavevector $k = 2\pi / \lambda$ are tied to frequency $\omega=ck$ by light speed c that Einstein's axiom fixes. So, a laser's (ω,ck)-vector must stay on its $\pm 45°$ baseline in Fig. 1a. Still, this c-axiom of Einstein is a pretty wild and crazy idea. For 100 years, students have been told, "Hey, it works! Trust me." But, a century later we ask, "Do axioms of relativity or of quantum theory *have* to seem so crazy? Do not such important subjects beg for more *intuitive* axioms?"

Axioms need Occam's razor, so named for William of Occam (circa 1300).[8] Occam said axioms get power by being as *irreducible* and as *self-evident* as possible. Einstein's axiom, as it is often stated using c-constancy of lightning flashes in all train-frames, is hardly self-evident. Also, flashes or pulses are multi-component monstrosities badly in need of an Occam-shave. Evenson's experiments point the way to the barbershop.

Suppose we restate Einstein's axiom as Evenson did: *all colors (frequencies) go the same speed* $c=\omega/k$ *(en-vacuo)*. If a pulse is Occam-shaved to one fundamental and irreducible spectral component then the resulting axiom is more self-evident. Even slightly non-c components would ruin Hubble images of 10^9-year old galaxies, and out-of-step color components suffer destructive interference unless light has linear dispersion $\omega = ck$. We need *equal* Doppler shifts for ω *and* k to uniquely define colors by either. Thus, "baseball" figures 1-2 follow a baseball-like rule: optical $\mathbf{K}$ vectors (ck,ω) must stay in their baselines $(\omega = \pm ck)$ and spacetime ray paths $(x = \pm ct)$ can only run on theirs. (Some results of non-light waves breaking these rules are noted in the appendix.)

A baseball diamond in per-spacetime

Fig. 1a resembles a baseball diamond. The 1st base vector $\mathbf{K_1} = (ck,\omega) = \omega(1,1)$ defines the *L-to-R* laser wave e^{ia} of phase $a = (kx - \omega t)$. The 3rd base vector $\mathbf{K}_3 = (-ck,\omega) = \omega(-1,1)$ defines the *R-to-L* laser wave e^{ib} of phase $b = (-kx - \omega t)$. Euler's identity lets them interfere in a Ψ wave sum.

$$\begin{aligned}\Psi = e^{ia} + e^{ib} &= e^{i\frac{a+b}{2}}[e^{i\frac{a-b}{2}} + e^{-i\frac{a-b}{2}}] \\ &= e^{i\frac{a+b}{2}}[\ 2\cos\left(\frac{a-b}{2}\right)\ ]\ = e^{ip}[2\cos g]\end{aligned} \tag{1}$$

The mean *phase* $p = (k_p x - \omega_p t)$ has base vector $\mathbf{K}_p$ that is half the sum of 1st and 3rd bases or half of 2nd base vector $\mathbf{K}_2 = \mathbf{K_1} + \mathbf{K}_3$. $\mathbf{K}_p$ points from home plate O to pitcher's mound P in Fig. 1b.

$$\mathbf{K}_p = (ck_p,\omega_p) = (\mathbf{K_1} + \mathbf{K}_3)/2 = \omega(0,1) \tag{2}$$

Factor e^{ip} is modulated by *group envelope* $[2\cos g]$ with $g = (k_g x - \omega_g t)$. Group base $\mathbf{K}_g$ is half the difference of 1st and 3rd bases and lies between pitcher's mound P and 1st or 3rd base in Fig. 1b.

$$\mathbf{K}_g = (ck_g,\omega_g) = (\mathbf{K_1} - \mathbf{K}_3)/2 = \omega(1,0) \tag{3}$$

Eqs. (1-3) define a standing wave $\Psi = e^{ip}[2\cos g] = e^{-i\omega t}[2\cos kx]$ plotted in spacetime with the *ct*-axis vertical in Fig. 1c. Real part $\mathrm{Re}\,\Psi = \cos\omega t[2\cos kx]$ is a dark blue sine curve whose crests and troughs trace dark green and blue checkerboard squares outlined by a white Cartesian spacetime (x,ct) lattice showing where $\mathrm{Re}\,\Psi$ is zero. Group base $\mathbf{K}_g = \omega(1,0)$ and phase base $\mathbf{K}_p = \omega(0,1)$ are each rescaled by $\frac{\pi}{\omega \cdot k}$ to become spacetime lattice vectors $\hat{\mathbf{e}}_x = \hat{\mathbf{K}}_g = (1,0)\frac{\lambda}{2}$ and $\hat{\mathbf{e}}_{ct} = \hat{\mathbf{K}}_p = (0,1)\frac{\lambda}{2}$. (Waves are shown in the appendix to make an (x,ct) lattice that mirrors their (ω,ck) grid.)

A spacetime baseball diamond (Build it and they will run.)

So far we have invoked 1970's CW (Continuous Wave) lasers, sharp in frequency but broadly delocalized in spacetime. Consider instead modern sub-femtosecond PW (Pulse Wave) lasers having trillions of frequency overtones. Overtones localize waves into spiked pulse trains even with a half-dozen octaves $\omega, 2\omega, 3\omega, \ldots, 6\omega$ that are used in Fig. 1d. The resulting pulses run the baselines of spacetime baseball diamonds, but any sign of a square lattice of Fig. 1c is phased out *except where pulses interfere*. (Note tiny square "bases" at pulse intersections in Fig. 1d.) The role and properties of wave interference are much more of a key to the foundation and framework of physics than is widely realized. The next few pages endeavor to show this.

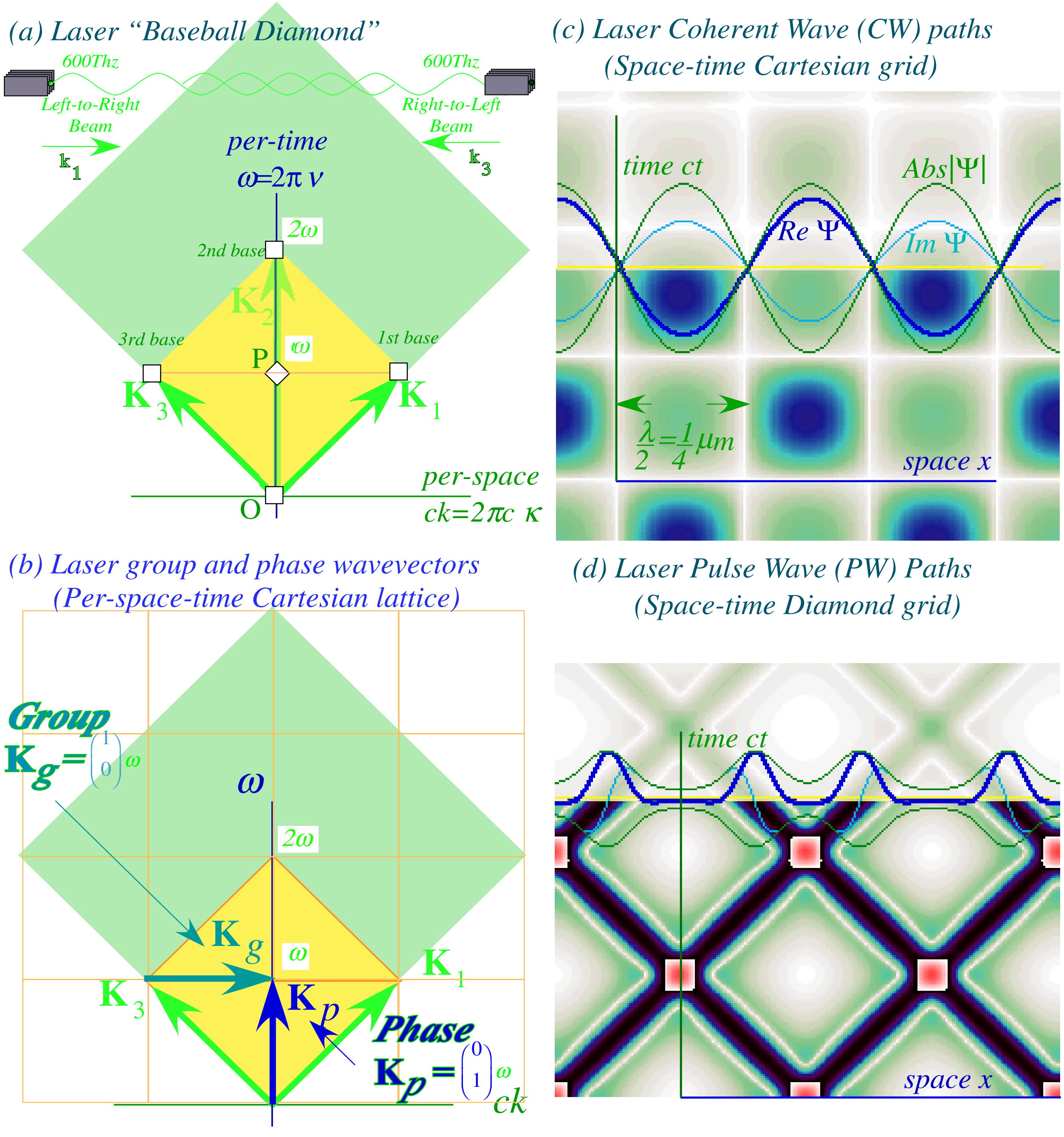


Fig. 1 Laser lab view of 600Thz CW and PW light waves in per-space-time (a-b) and space-time (c-d).

Comparing per-spacetime *vs.* spacetime shows wave-pulse duality. It is a space-and-time analog of a spatial real-lattice *vs.* reciprocal-lattice in X-ray crystallography. A per-spacetime baseball diamond in Fig. 1a yields a continuous wave (CW) spacetime Cartesian phase-*vs*-group-wave lattice in Fig. 1c, but an $\omega(m,n)$-overtone sum of Cartesian per-spacetime lattice vectors $m\mathbf{K}_g + n\mathbf{K}_p$ in Fig. 1b yields a spacetime baseball diamond pulse wave (PW) lattice in Fig. 1d. As shown in the appendix, it is quite revealing to put *k*-*vs*-*ω* and *t*-*vs*-*x* plots right on top of each other, so a wave function and its Fourier transform coexist on the same piece of graph paper.

Relativistic baseball geometry: Doppler rules

An observant atom, going right-to-left at a high speed u in a spacetime grid like Fig. 1c, might see emerald green turn into garish purple due to Doppler shifts. The grid itself undergoes a squeezing in Fig. 2c. Geometry in Fig. 2a-b derives this by considering Doppler baseline shifts.

The *R-to-L* beam, along which the atom moves, is Doppler red-shifted by some factor r, and the approaching *L-to-R* beam will be blue-shifted by an inverse factor $b=1/r$. The inverse relation of r and b is needed for time-reversal symmetry. If my *1Thz* transmitter resonates your *2Thz* receiver as we approach $(b=2)$ then my *1Thz receiver* responds to your *2Thz transmitter* as we depart $(r=1/2)$. For astronomical b, a logarithmic *rapidity* parameter $\rho=\ln b$ is conventional and convenient. In Fig. 2a-b, we let a *2:1* ratio $b=2=e^{\rho}$ and $r=1/2=e^{-\rho}$ shift our bases.

Fig. 1a lets the 1st base vector $\mathbf{K}_1=\varpi(1,1)$ of beam-*L-to-R* double in length $(b=2)$ to $\mathbf{K}_1'=\omega_1(1,1)=\varpi(b,b)=\varpi(e^{\rho},e^{\rho})$ as 3rd base vector $\mathbf{K}_3=\varpi(-1,1)$ of beam-*R-to-L* halves $(r=1/2)$ to $\mathbf{K}_3'=\omega_3(-1,1)=\varpi(-r,r)=\varpi(-e^{-\rho},e^{-\rho})$. So, group-phase bases (2-3) in Fig. 1 transform to (4-5).

$$\mathbf{K}_g'=(ck_g',\omega_g')=\frac{(\mathbf{K}_1'-\mathbf{K}_3')}{2}=\varpi\frac{(e^{\rho}+e^{-\rho},e^{\rho}-e^{-\rho})}{2}=\varpi(\cosh\rho,\sinh\rho)=\varpi(\frac{5}{4},\frac{3}{4}) \qquad (4)$$

$$\mathbf{K}_p'=(ck_p',\omega_p')=\frac{(\mathbf{K}_1'+\mathbf{K}_3')}{2}=\varpi\frac{(e^{\rho}-e^{-\rho},e^{\rho}+e^{-\rho})}{2}=\varpi(\sinh\rho,\cosh\rho)=\varpi(\frac{3}{4},\frac{5}{4}) \qquad (5)$$

Vectors (4-5) and geometry in Fig. 2 give a Lorentz[9]- Einstein[10] transformation for per-spacetime-(ck,ω) *and* for spacetime-(x,ct). Phase-sum base $\mathbf{K}_p'$ and group-difference base $\mathbf{K}_g'$ are half-diagonals of a Doppler-shifted diamond spanning a (ck,ω) grid[11] in Fig. 2b and a Minkowski (x,ct) grid made in Fig. 2c by CW interference. The laser lab time-*ct*-axis lies on primitive lattice vector $\hat{\mathbf{e}}_t=\hat{\mathbf{K}}_p'=(\frac{3}{4},\frac{5}{4})\frac{\lambda}{2}$ from (5) and the lab space-*x*-axis lies on $\hat{\mathbf{e}}_x=\hat{\mathbf{K}}_g'=(\frac{5}{4},\frac{3}{4})\frac{\lambda}{2}$ from (4). Fig. 2d shows how lab pulse waves (PW) look to the atom. (Note tiny Minkowski diamond "bases.")

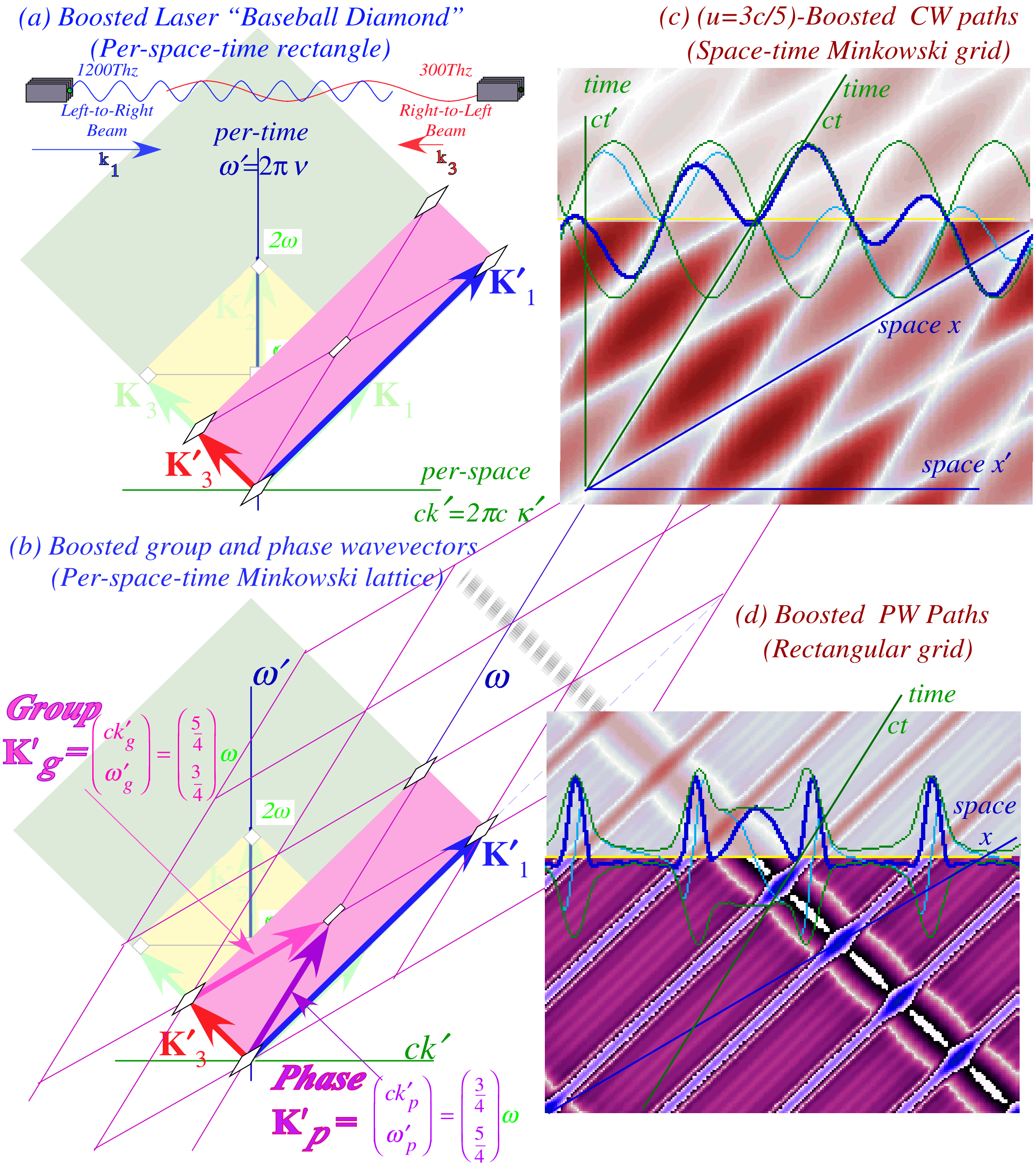
Consider the slope $\omega_g'/ck_g'=\frac{3}{5}$ of $\mathbf{K}_g'$. That ratio is wave group velocity (in c units) as seen in the atom frame where the laser lab and its "standing" wave go by at velocity u. So, ω_g'/ck_g' is set equal to u/c, the atom-observed speed of the group wave seen to be standing still by the lab.

$$\beta\equiv\frac{u}{c}=\frac{\omega_g'}{ck_g'}=\frac{\sinh\rho}{\cosh\rho}=\tanh\rho\xrightarrow[u<<c]{}\rho\to 0 \qquad (6a)$$

So $b=2$ gives $\frac{u}{c}=\frac{3}{5}$, and low u/c <u>is</u> rapidity ρ. Also, phase velocity ω_p'/ck_p' is an *inverted* $\frac{c}{u}=\frac{5}{3}$.

$$\frac{1}{\beta}=\frac{c}{u}=\frac{\omega_p'}{ck_p'}=\frac{\cosh\rho}{\sinh\rho}=\coth\rho\xrightarrow[u<<c]{}\frac{1}{\rho}\to\infty \qquad (6b)$$

This connects coherent wave (CW) relativity to the Newtonian-corpuscular or pulse-wave (PW) relativity of Einstein. *1st*-order Doppler color shifts are *first* in CW theory. Since pulses are white, PW theory tends to deal first with *2nd*-order factors like the Einstein time dilation factor γ.



$$\text{Group } \mathbf{K}'_g = \begin{pmatrix} ck'_g \\ \omega'_g \end{pmatrix} = \begin{pmatrix} \frac{5}{4} \\ \frac{3}{4} \end{pmatrix}\omega \qquad \text{Phase } \mathbf{K}'_p = \begin{pmatrix} ck'_p \\ \omega'_p \end{pmatrix} = \begin{pmatrix} \frac{3}{4} \\ \frac{5}{4} \end{pmatrix}\omega$$

Fig.2 Atom view of 600Thz CW and PW light waves in per-spacetime (a-b) and space-time (c-d) boosted to u=3c/5.

2^{nd}-order Lorentz contraction $1/\gamma$ and Einstein time dilation factor γ are well known PW results.

$$\gamma \equiv \cosh\rho = \frac{1}{\sqrt{1-\tanh^2\rho}} = \frac{1}{\sqrt{1-u^2/c^2}} \xrightarrow[u<<c]{} 1+\frac{\rho^2}{2} = 1+\frac{u^2}{2c^2} \qquad (7)$$

But, factor $\gamma \equiv \cosh\rho$ and asimultaneity factor $\beta\cdot\gamma \equiv \sinh\rho$ arise more naturally in CW results (4-5).

$$\beta\cdot\gamma \equiv \tanh\rho\cdot\cosh\rho = \sinh\rho = \frac{u/c}{\sqrt{1-u^2/c^2}} \xrightarrow[u<<c]{} \rho+\frac{\rho^3}{6} = \frac{u}{c}+\frac{u^3}{6c^3} \qquad (8)$$

The Doppler factor for blue-shift $b = e^{\rho}$ (red-shift $r = e^{-\rho}$) is the sum (difference) of (7) and (8).

$$b = e^{\rho} = \cosh\rho + \sinh\rho \;= \sqrt{\frac{1+u/c}{1-u/c}} \xrightarrow[u<<c]{} 1+\rho = 1+\frac{u}{c} \qquad (9a)$$

$$r = e^{-\rho} = \cosh\rho - \sinh\rho = \sqrt{\frac{1-u/c}{1+u/c}} \xrightarrow[u<<c]{} 1-\rho = 1-\frac{u}{c} \qquad (9b)$$

Components (4-5) of $\mathbf{K}'_g$ and $\mathbf{K}'_p$ make a two-by-two Lorentz matrix to act on the atom's readings of spacetime (x', ct') or per-spacetime (ck', ω') and yield lab readings (x, ct) or (ck, ω), respectively.

$$\begin{aligned} x\; &= x'\cosh\rho + ct'\sinh\rho \\ ct &= x'\sinh\rho + ct'\cosh\rho \end{aligned} \qquad (10a) \qquad \begin{aligned} ck &= ck'\cosh\rho + \omega'\sinh\rho \\ \omega\; &= ck'\sinh\rho + \omega'\cosh\rho \end{aligned} \qquad (10b)$$

Thus, Occam's razor logic and Evenson's laser metrology lead to wave coordinates that have logical and empirical sharpness. Optical metrology underlies a Global Positioning System[12] (GPS), an enormously precise spacetime coordinate system, made not of steel but of waves. It is notable that the greatest precision arises not from concrete and absolute meter sticks but rather from the ethereal and inherently relative action of optical *wave interference*.

Geometric means to Einstein invariants

As spacetime coordinates (x, ct) and per-spacetime wave parameters (ck, ω) vary in (10) from one frame to another, Einstein reassures us with quantities that do not vary. One such quantity is his spacetime invariant that is widely known as proper time τ or "age."

$$(c\tau)^2 = (ct)^2 - x^2 = (ct')^2 - x'^2 \qquad (11)$$

The inverse or per-spacetime CW invariant is called proper frequency ϖ or "rate of aging."

$$(\varpi)^2 = \omega^2 - (ck)^2 = \omega'^2 - (ck')^2 \qquad (12)$$

In the CW results (4-5), ϖ is the *geometric* mean $\sqrt{\omega_1\omega_3} = \varpi$ of the blue-shifted laser frequency $\omega_1 = \varpi b$ and red-shifted frequency $\omega_3 = \varpi r$. Time-reversal symmetry $(br = 1)$ makes baseball diamonds in Fig. 2b maintain area $2\varpi^2 = 2\omega_1\omega_3$ at any speed u. In Fig. 3a-b Euclid's semi-circle has vertical radius SP equal to the *arithmetic* mean $[\omega_1 + \omega_3]/2 = \langle\omega\rangle$ of laser frequencies. SP is the ordinate $\langle\omega\rangle = \varpi\cosh\rho$ of pitcher's mound P on the tipped ω -axis. The semi-circle connects a geometric mean point-ϖ on the vertical ω' -axis to Doppler shifted $\omega_3 = \varpi r$ and $\omega_1 = \varpi b$ on the atom's ck' -axis. Point-P lies on the ϖ -hyperbola (12) at $\varpi(\sinh\rho, \cosh\rho)$ as derived by (5). Mean difference $[\omega_1 - \omega_3]/2 = \langle\omega_-\rangle = \varpi\sinh\rho$ is the horizontal component OS of pitcher's mound P .

Two equilateral hyperbolas (12) are plotted in Fig. 3c using Euclid's construction of a geometric mean. It is a *contact* construction whereby each hyperbola is an envelope of contact tangents. Each axis $\omega, \omega', \omega''$ marks a contact point P, P', P'' such as the *n=1* points in Fig. 3c on tangents $LP, L'P', L''P''$. Each tangent at hyperbolic radius $\omega' = n\varpi$ is a grid line parallel to the ck' axis in its Minkowski per-spacetime (ck', ω') frame. Concentric $(n\varpi)$-hyperbolas *(n=1,2,)* are per-spacetime grid markers mirroring Einstein's spacetime metric (11). Per-spacetime hyperbolas are invariant dispersion functions $\omega_n(k) = \sqrt{n^2\varpi^2 + c^2k^2}$. Each belongs to a Planck quantum $(n\hbar\varpi)$.

Keep the phase and walk with Planck

Spacetime and per-spacetime together make an all-important invariant called *wave phase*.

$$\Phi = -\varpi\tau = kx - \omega t = k'x' - \omega' t' \qquad (13)$$

Phase is a dimensionless invariant for *all* waves, not just light. Each reading of a wave phase is a proper quantity, a clock read-out if you will, and quite invariant to Lorentz-frame choice. Phase is related to *action*, but that's getting ahead of our story. First, wave phase angular *velocity* ω_p has to be related to the classical energy of Newtonian mechanics and to Galilean momentum[13].

We need Max Planck's 1900 axiom $E=n\hbar\omega$ in his theory of low-T light.[14] Like Einstein's 1905 light speed axiom $\omega = ck$, we take $E=n\hbar\omega$ literally; classical energy E *is* $\hbar\omega_p$ of (5) scaled by n arising from an underlying *phase* wiggle $\omega_p = n\varpi \cosh\rho$ due to n of Planck's quanta $\hbar\varpi$.

In 2005, this is not regarded as such a wild and crazy idea. But, Newton would likely be skeptical that wiggling of an invisible wave phase underlies his mechanics. Indeed, Planck himself regarded $E=n\hbar\omega$ with remorse and tried to discard it![15] Oscillator energy, *linear* in ω, seems in conflict with the classical oscillator energy $E = \frac{1}{2} MA^2\omega^2$ that is *quadratic* in both amplitude A and frequency ω. In 1900, no one knew of the $\omega^{-1/2}$ dependence of a quantum field amplitude A or its relation to Plank's n-quantum number in eq. (20) below.

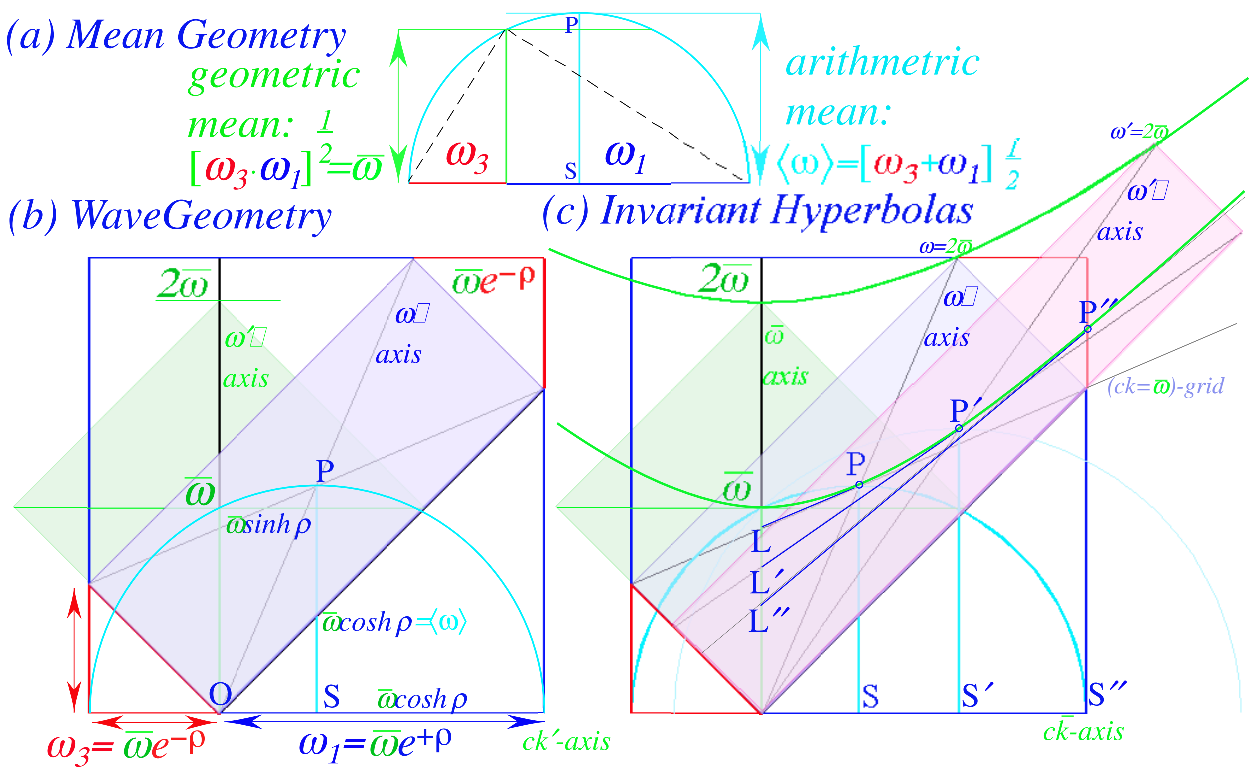


Fig.3 Euclidean construction (a) of relativistic wave geometry (b) and hyperbolic invariants (c).

Einstein did not share Planck's remorse. In his 1905 developed relations consistent with $E = \hbar n\omega$.[16] Still, he did not use it in quite the literal sense that follows here. Let's see how things simplify if Planck's energy $E = \hbar\omega_p$ in $\hbar$-units equals overall wave phase rate (5).

$$E = \hbar\omega_p = \hbar n\varpi \cosh\rho \xrightarrow[u<<c]{} \hbar n\varpi + \tfrac{1}{2}\frac{\hbar n\varpi}{c^2}u^2 \tag{14}$$

The resulting low-u energy $E \approx A + \frac{1}{2}Bu^2$ looks like Newton's $KE = \frac{1}{2}Mu^2$ apart from a constant first term $A = Bc^2 = \hbar n\varpi$. It's then tempting to set wave factor $B = \hbar n\varpi / c^2$ to Newton's mass M.

$$E = \hbar n\varpi \cosh\rho = \frac{Mc^2}{\sqrt{1-u^2/c^2}} \xrightarrow[u<<c]{} Mc^2 + \tfrac{1}{2}Mu^2 \tag{15}$$

Then wave $\hbar\omega_p$ is Einstein's 1905 energy. Wavevector $ck_p = n\varpi\sinh\rho$ of (5) times $\hbar$/c is exactly DeBroglie[17] momentum p or by (8), its Galilean approximation $p=Mu$. CW light tells us a lot!

$$p \equiv \hbar k_p = \frac{\hbar n\varpi}{c}\sinh\rho = \frac{Mu}{\sqrt{1-u^2/c^2}} \xrightarrow[u<<c]{} Mu + \frac{Mu^3}{6c^3} \approx Mu \tag{16}$$

When ω_1 and ω_3 CW beams collide, their interference makes a coordinate grid in Fig. 2c. Its rest frame, shown in Fig. 1c, is wherever both beams appear with the same color $\varpi = \sqrt{\omega_1\omega_3}$. Light beam ω_1 or ω_3 has no rest frame or mass by itself since each one's proper frequency is zero for baselines $(\omega = \pm ck)$, yet together they form optical modes of proper frequency ϖ , energy $n\hbar\varpi$, and a very tiny mass $n\hbar\varpi/c^2$. But, one may ask why simple *wave* optics leads directly to general properties (15) and (16) of relativity and quantum mechanics of a *particle*, and how might a box of counter-propagating green light *waves* act like *particles* of total mass $M=\hbar n\varpi/c^2$?

Quantizing everything

The short answer to the first question is that particles are waves, too, and so forced by Lorentz symmetry to use available hyperbolic invariants $\omega^2 - (ck)^2 = (Mc^2/\hbar)^2$ for dispersion. To answer the second question entails more loss of classical innocence as we again apply Occam's razor to cut semi-classical laser waves down to a single field quantum $\hbar\varpi$ or *photon*. So the second short answer is that waves are particles, too, even for optical dispersion $(\omega^2 - (ck)^2 = 0)$.

Classical laser waves use Maxwell classical fields **E** and **B** or else vector potential **A** and its time derivative $\dot{\mathbf{A}} = -\mathbf{E}$ to make complex Fourier amplitudes $\mathbf{a}_k$ and $\mathbf{a}_k^*$ for each mode that is an oscillator of frequency $\omega = \pm ck$. Classical field-theory amplitudes, fixed by initial values, let us predict wave magnitude and phase everywhere. The "laser wave" in (1) has zero amplitudes $\mathbf{a}_k^* = 0 = \mathbf{a}_k$ for all except two *600Thz* plane waves $\mathbf{K}_1$ and $\mathbf{K}_3$ for which $\mathbf{a}_k^* = 1 = \mathbf{a}_k$.

Classical wave variables k and ω were known to be discrete or "quantized" by boundary conditions. Bohr's k-quantization $k_m = m\frac{2\pi}{L} = mk_1$ gives momenta $p_m = \hbar k_m = mp_1$ by (16) with $m=0,\pm1$ $\pm2,\ldots$ quantum numbers.[18] Quantized rotor energy $E_m = m^2E_1$ results by (15) with $E_1 = p_1^2/2M$.

Heisenberg[19] showed quanta p_m or E_m arise from eigenvalues (literally "own-values") of matrix operators $\mathbf{p}$ or $\mathbf{H}$ whose eigenvectors ("own-vectors") $|p_m\rangle$ or $|E_m\rangle$ may be superimposed.

$$|\Psi\rangle = \psi_1|E_1\rangle + \psi_2|E_2\rangle + \psi_3|E_3\rangle + \qquad (17)$$

(Dirac's bra-ket[20] notation came later.) Allowing things to be at (or in) m places (or states) lets mean values $\bar{E} = \langle\Psi|\mathbf{H}|\Psi\rangle$ range continuously from lowest quantum levels E_1 to the highest E_m.

$$\bar{E} = \langle\Psi|\mathbf{H}|\Psi\rangle = |\psi_1|^2 E_1 + |\psi_2|^2 E_2 + |\psi_2|^2 E_2 + \qquad (18)$$

For classicists, the notion that each multiple-personality-k has a probability $|\psi_k|^2$ seems, if not wild and crazy, then at least dicey, as in Einstein's skeptical quote, "God does not play dice…" [21]

But, superposition is really an idea borrowed from classical wave mechanics. Waves add and interfere making them ultra-sensitive to relative position and velocity, a *first* order sensitivity that leads elegantly to relativistic coordinate and kinematic relations (15-16) by geometry (5) of optical wave phase variables k, ω, x, and t. But, what about the optical wave *amplitudes* $\mathbf{A}$ or $\mathbf{E}$? Amplitude "2" of wave (1) is set arbitrarily. Without Planck, its value is "un-quantized."

Quantum amplitude vs. quantum phase

Now we know *amplitudes* $\mathbf{A}$ or $\mathbf{E}$ have "quantizing" operators, too. Mode amplitude $\mathbf{a}_k$ or $\mathbf{a}_k^*$ in classical energy $\sum \omega_k^2 \mathbf{a}_k^* \mathbf{a}_k$ is replaced by oscillator boson operator $\mathbf{a}_k$ or $\mathbf{a}_k^\dagger$ in a quantum field Hamiltonian $\mathbf{H} = \Sigma\hbar\omega_k(\mathbf{a}_k^\dagger \mathbf{a}_k + \frac{1}{2})$ whose eigenstates $|N_1 N_2 \cdots N_k\rangle$ have exact quantized photon numbers $\langle \mathbf{a}_k^\dagger \mathbf{a}_k \rangle = N_k$, definite amplitudes for each mode-k_m, but totally *un*certain field phases.

Field quantization is often called "second quantization" to distinguish k_m *mode* numbers m, that seem purely classical for light, from its "purely quantum" *photon* numbers $n = N_{k_m}$. This may be a prejudice that waves (particles) are usual (unusual) for light but unusual (usual) for matter. Einstein seems not to have shared such terminology or prejudice. Yet, Bose-Einstein theory of photon symmetry remains well established in laser physics and quantum optics today.

Model micro-laser states are *coherent* states $|\alpha\rangle = \Sigma_n \psi_n |n\rangle$ made of single-mode eigenstates $|n\rangle = (\mathbf{a}_1^\dagger)^n |0\rangle$ with amplitudes $\psi_n = \alpha^n e^{-\alpha^2/2} / \sqrt{n!}$. Variable $\alpha = x + ip = |\alpha| e^{i\Phi}$ is average mode phase, and $(x = \mathrm{Re}\,\alpha, p = \mathrm{Im}\,\alpha)$, rescaled by a quantum field factor f, are field averages $(\langle A\rangle, \langle\dot{A}\rangle = -\langle E\rangle)$.

$$\langle\alpha|A|\alpha\rangle = \langle A\rangle = (\alpha + \alpha^*) f = (\alpha + \alpha^*)\sqrt{\frac{\hbar}{2\varepsilon_0 \omega V}} \qquad (19)$$

Amplitude factor f makes Planck's $\bar{E} = \hbar\omega\bar{n}$ equal Maxwell field energy $\bar{E} = \bar{U}\cdot V$.

$$\langle U\rangle V = 2\varepsilon_0 \omega^2 V \langle A^2\rangle = \hbar\omega|\alpha|^2 = \hbar\omega\bar{n} \qquad (20)$$

A fundamental laser mode in a $0.25\mu m$ cubic cavity (See E-wave sketched in one strip of Fig. 1c.) has green light with $\hbar\varpi = 4\cdot 10^{-19}\,Joule$ or $2.5eV$ per photon. The average photon number $\bar{n} = |\alpha|^2 = 10^{10}$ models a laser with mean energy $\bar{E} = \bar{U}\cdot V = \hbar\varpi\bar{n} = 4.0\,nanoJ$ in a volume $V = (\frac{1}{4}\mu m)^3$. Photon number uncertainty $\Delta n = |\alpha| = 10^5$ varies inversely to phase uncertainty $\Delta\Phi = \pi/\alpha \sim 3\cdot 10^{-5}$.

Amplitude expectation value $\langle n|A|n\rangle$ is zero for $|n\rangle$ states due to *in*coherence of phase, but number value $\langle n|\mathbf{a}_k^\dagger \mathbf{a}_k|n\rangle = n$ is exact as is proper frequency ϖn due to the phase factor $(e^{-i\varpi t})^n$ of $(\mathbf{a}_1^\dagger)^n$. For *any* volume *V*, these $(n = 10^{10})$-photons have energy $E = \hbar\omega n$ or mass $M = E/c^2 = 10^{-25}\,kg$ equal to that of 59.79 H-atoms, but it's not "real" mass. (Real $e + \bar{e}$ pair-creation means raising ϖ from *600Thz* to $m_e c^2/h$ or *100MegaThz*.) Nevertheless, "real" $10^{-25}\,kg$ and "optical" $10^{-25}\,kg$ share a hyperbola 10^{10} quanta above the *n=1* hyperbola in Fig. 3c. A coherent-state $|\alpha = 10^5\rangle$ also has a mass $M = 10^{-25}\,kg$ but with uncertainty $\Delta M = 10^{-30}\,kg$. Its *phase* uncertainty $3\cdot 10^{-5}$ is low enough to plot grids like Fig. 1c or Fig. 4a. A low-α state (Fig. 4c) has too few photon counts-per-grid to plot sharply. Photon eigenstate $|n\rangle$ is a wash even for high *n* since $\Delta n = 0$ has $\Delta\Phi = \infty$. (Fig. 4d)

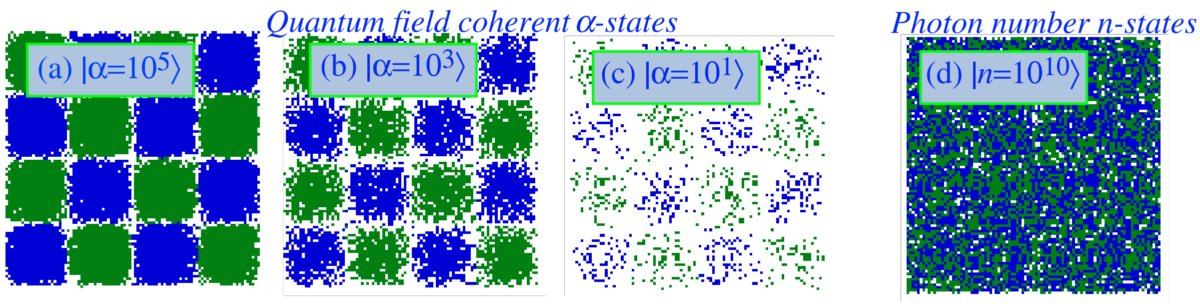


Fig. 4 Simulated spacetime photon counts for coherent (a-c) and photon-number states (d).

What (or which) mass?

Mass and force are quite enigmatic concepts in classical physics. So, it is interesting if an Einstein-Planck wave frequency-energy-mass equivalence relation derived in (15) ascribes rest mass M_{rest} to an electromagnetic phase rate ϖ. Could all our "*M*-stuff" be due to a "wiggle" ϖ?

$$M_{rest} = \hbar n\varpi / c^2 \tag{21}$$

While Einstein's rest-mass is frame invariant, momentum-mass M_p varies with frame velocity *u*. Like Galileo's original definition, this mass M_p is a ratio p/u of momentum (16) and velocity.

$$\frac{p}{u} \equiv M_p = \frac{\hbar n\varpi}{cu}\sinh\rho = M_{rest}\Big/\left(1 - u^2/c^2\right)^{1/2} \xrightarrow[u<<c]{} M_{rest}$$

Frame velocity *u* is wave group velocity as noted in (6a). The Euclid mean construction of Fig. 3c shows *u* is the slope of the dispersion tangent. A derivative of energy (15) verifies this.

$$V_{group} = \frac{d\omega}{dk} = \frac{dE}{dp} = \frac{c^2 p}{E} = u \tag{22}$$

What Newton defines is *effective* mass M_{eff}, a ratio $\dot{p}/\dot{u}$ of *change* of momentum and velocity.

$$\frac{F}{a} \equiv M_{eff} \equiv \frac{dp}{du} = \frac{\hbar dk}{dV_{group}} = \hbar\Big/\frac{d}{dk}\frac{d\omega}{dk} = \hbar\Big/\frac{d^2\omega}{dk^2} = M_{rest}\Big/\left(1 - u^2/c^2\right)^{3/2} \xrightarrow[u<<c]{} M_{rest} \tag{23}$$

Wave *effective* mass enters Newton's $F = M_{eff}a$. As *u* nears *c*, M_{eff} grows even faster than M_p, but at low speed, both equal the invariant M_{rest}. Relativity and quantum theory keep absolute frame-invariant quantities behind the scenes. What we *observe* are relative frame-dependent values.

What's observable? The beats go on unphased

Quantum observables derive from probabilities $\psi^*\psi$ that are wave amplitude squares. Squaring a wave like eq. (1) eliminates its phase factor $e^{i(a+b)/2}$, and only group functions $\cos^2(\frac{a-b}{2})$ of *differences* of 1st and 3rd base frequencies or *k*-vectors, $\omega_1-\omega_3$ or k_1-k_3, remain observable.

Group frequency $\Delta\omega=\omega_1-\omega_3$ is zero in rest frame Fig. 1c so it is a stationary state. In all other frames like Fig. 2c we observe motion and $\psi^*\psi$ is not at rest. Fourier sums of *m=3* or more components $\Psi=a_1e^{i(k_1x-\omega_1t)}+a_2e^{i(k_2x-\omega_2t)}+a_3e^{i(k_3x-\omega_3t)}+...$ have multiple beats in $\Psi^*\Psi$ as in Fig. 3d.

$$P=\Psi^*\Psi=\sum a_i{}^*a_je^{i(\Delta k_{ij}x-\Delta\omega_{ij}t}$$

With $m(m-1)/2$ observed *difference* $\Delta\omega_{ij}=\omega_1-\omega_j$ or *beat* notes, P cannot rest in any frame. We see relative quantities, differences or derivatives, but absolute Ψ-phases hide *until waves of two quantum objects interfere*. Then a new absolute phase is a sum of individual phases while the relative difference in phases adds new beats. That we are limited to only pick up beats[22] or relative frequency is perhaps a quantum version of Einstein's mythical saw, "It's *all* relative."

Total phase gives total energy and momentum, yet it is their *change* that we measure. The phase part of a quantum wave is like a carrier frequency of radio waves, fast and undetected, and the group part is like the audible signal, slower and detected as resonant beats between the carrier and receiver. The observed emerald green of our *4.0* nJ laser, is really a $600THz$ $\varpi=\omega_n-\omega_{n-1}$ *beat note* from coherent interference between each neighboring pair in 10^5 or so levels lying around a carrier or expected total phase frequency $\bar{n}\varpi$ of *6.0 TeraTera Hz* at the $\bar{n}=10^{10}$ quantum level.

Phase is invariant to choice of reference frame. Classical phase invariance goes back to Poincare's action differential *dS* (24a). Called *Poincare's invariant*, it comes from the Legendre transformation (24b) relating a classical Hamiltonian $H(p,x)$ to a classical Lagrangian $L(\dot{x},x)$.

$$dS=Ldt=pdx-Hdt \qquad (24a) \qquad\qquad L=p\dot{x}-H \qquad (24b)$$

Invariant *dS* is integrable if momentum p and Hamiltonian H satisfy Hamilton-Jacobi equations.

$$\frac{\partial S}{\partial x}=p\equiv\frac{\partial L}{\partial\dot{x}} \qquad (25a) \qquad\qquad \frac{\partial S}{\partial t}=-H \qquad (25b)$$

Planck-DeBroglie axioms give semi-classical action S in terms of phase Φ of quantum wave Ψ.

$$\hbar\,d\Phi=dS=Ldt=\hbar k\,dx-\hbar\omega\,dt \quad \text{where:} \quad \Psi\approx e^{i\Phi}=e^{i\int Ldt/\hbar} \qquad (26)$$

This Hamilton-Dirac-Feynman[23] action-phase relation in free space-time is plane wave phase.

$$S/\hbar=\Phi=kx-\omega t. \qquad (27)$$

Hamilton's velocity equation (28a) is related to the definition (28b) of wave group velocity.

$$\dot{x}=\frac{\partial H}{\partial p} \qquad (28a) \qquad\qquad u=\frac{\partial\omega}{\partial k} \qquad (28b)$$

Hamilton's "force" equation (29a) relates to wave refraction (29b) by spatial gradient.

$$\dot{p}=-\frac{\partial H}{\partial x} \qquad (29a) \qquad\qquad \dot{k}=-\frac{\partial\omega}{\partial x} \qquad (29b)$$

Which is first? Classical chicken or quantum egg?

An x-derivative (25a) used on semi-classical wave (26) resembles a momentum-**p**-operator definition familiar to quantum mechanics.

$$\frac{\partial}{\partial x}\Psi \approx \frac{i}{\hbar}\frac{\partial S}{\partial x}e^{iS/\hbar} = \frac{i}{\hbar}p\Psi \qquad (30a) \qquad\qquad \frac{\hbar}{i}\frac{\partial}{\partial x}\Psi = p\Psi \qquad (30b)$$

The time derivative is similarly related to the Hamiltonian operator. The H-J-equation (25b) makes this appear to be the Schrodinger[24] time equation.

$$\frac{\partial}{\partial t}\Psi \approx \frac{i}{\hbar}\frac{\partial S}{\partial t}e^{iS/\hbar} = -\frac{i}{\hbar}H\Psi \qquad (31a) \qquad\qquad i\hbar\frac{\partial}{\partial t}\Psi = H\Psi \qquad (31b)$$

Putting these in a classical Hamiltonian $H=p^2/2M+V(x)$ gives the usual Schrodinger equation.

But, this approximation ignores relativity. Potential energy $V(x)$ has no momentum part. Time derivative $i\hbar\frac{\partial}{\partial t}$ is 1st order while spatial derivative $-\frac{\hbar^2}{2M}\frac{\partial^2}{\partial x^2}$ is 2nd order. So, the relativity-quantum connection is compromised. Also, the $\approx$ sign on wave (26) warns of ignoring amplitude $|\Psi| = \sqrt{\Psi^*\Psi}$ thereby amputating essential quantum anatomy that Planck's axiom needs.

The path of extreme phase is the one true way

There are many such attempts to derive quantum theory from classical mechanics. All appear quite futile without the wave mechanics and relativity of phase variation. A quantum energy or frequency $\omega=E/\hbar$ is a negative time derivative (25b) of the phase $\Phi=S/\hbar$ and a very fundamental quantity due to its relativistic invariance. Lagrangian L (24b) or action $S=\int L dt$ are thus more fundamental than the Hamiltonian H or the momentum p are by themselves.

Indeed, classical mechanics of action was used in Bohr's first quantum theory and in Einstein-Bohr-Kellar quantization. This has since seen a revival in quantum chemistry. Relating Poincare's classical invariant (24) to quantum phase invariance is a very important step.

Later Feynman imagined families of classical paths fanning out like rays from each spacetime point. Normal to the classical momentum $\mathbf{p}=\nabla S$ (25a) of each ray, are wavefronts of constant phase Φ or action S. Then, according to a quantum rendition of Huygen's principle, new wavefronts are continuously built as in Fig. 5 through interference from "the best" of all the little wavelets emanating from a multitude of source points on a preceding wavefront.

The "best" are so-called *stationary-phase* rays that are extremes in phase and thereby satisfy *Hamilton's Least-Action Principle* requiring that $\int L dt$ is minimum for "true" classical trajectories. This in turn enforces Poincare' invariance by eliminating, by dephasing, any "false" or non-classical paths because they do not have an invariant (and thereby stationary) phase and cancel each other in a cacophonous mish-mash of mismatched phases. Each Huygen wavelet is tangent to the next wavefront being produced. That contact point is precisely on the ray or true classical trajectory path of minimum action and on the resulting wavefront.

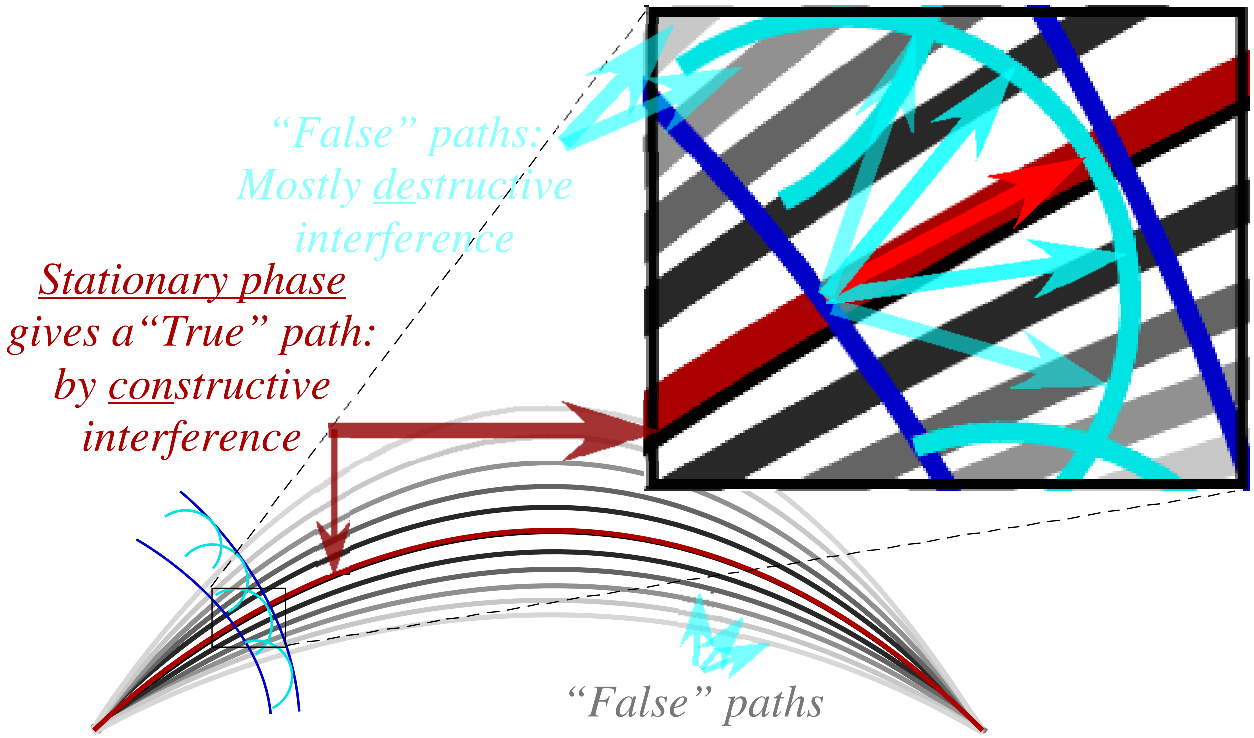


Fig. 5 Quantum waves interfere constructively on "True" path but mostly cancel elsewhere.

Huygen's generation of each wavefront by a preceding one corresponds to a quite old and beautiful geometric operation called a *contact transformation*. This is another place where wave geometrical petal meets quantum metal. Classical contact transformations correspond to the all-important quantum unitary transformations. Such transformations often hide a subtle geometry.

The simplest type of contact transformation is a Legendre transformation in which the contacting curves are straight lines. The transformation (24) of a Lagrangian function $L(q,u)$ of coordinate $q=x$ and velocity $u=\dot{x}$ to a Hamiltonian function $H(q,p)$ of q and momentum p is a familiar contact transformation in physics. Its relativistic algebra and geometry is given now.

Making contact with the phase

If a plane wave phase $\Phi = kx\text{-}\omega t$ has a rest frame where $x=0=k$, it then reduces to $-\mu\tau$, a product of proper frequency $\mu=n\varpi$ and proper time $t=\tau$. Invariant differential $d\Phi$ is reduced, as well, using the Einstein-Planck mass-energy-frequency equivalence relation (21) to

$$d\Phi = kdx - \omega dt = -\mu\ d\tau = -(Mc^2/\hbar)\ d\tau. \qquad (32)$$

τ-Invariance (11) or time dilation (7) gives proper $d\tau$ in terms of velocity $u=\frac{dx}{dt}$ and lab dt.

$$d\tau = dt\ \sqrt{(1-u^2/c^2)}\ ) = dt\ \mathrm{sech}\ \rho \qquad (33)$$

Combining definitions for action $dS=Ldt$ (24a) and phase $dS = \hbar d\Phi$ (26) gives Lagrangian L.

$$L = -\hbar\mu\tau = -Mc^2\sqrt{(1-u^2/c^2)} = -Mc^2 \mathrm{sech}\ \rho \qquad (34)$$

Fig. 6 has plots of both Lagranian L and Hamiltonian H using units for which $c=1=M$.

The *relativistic matter Lagrangian* in Fig. 6b is a circle. Three L-values L, L', and L'' in Fig. 6 are Legendre contact transforms of the three H-values H, H', and H'' on the Hamiltonian hyperbola in Fig. 6a. Abscissa p and ordinate H of a point P in plot (a) gives negative intercept $-H$ and slope p of the tangent contacting the transform point Q in plot (b). This works *vice-versa*, too.

The contact geometry exposes structure of wave-action-energy mechanics. Many students do not distinguish Newton-Lagrange kinetic energy $L = \frac{1}{2} Mu^2$ from $H = p^2 / 2M$ of Hamilton since putting in $P = Mu$ seems trivial. But, hyperbola H and circle L only follow a similar parabola at low speed $u<<c$. As u approaches c their considerable differences expose themselves in Fig.6.

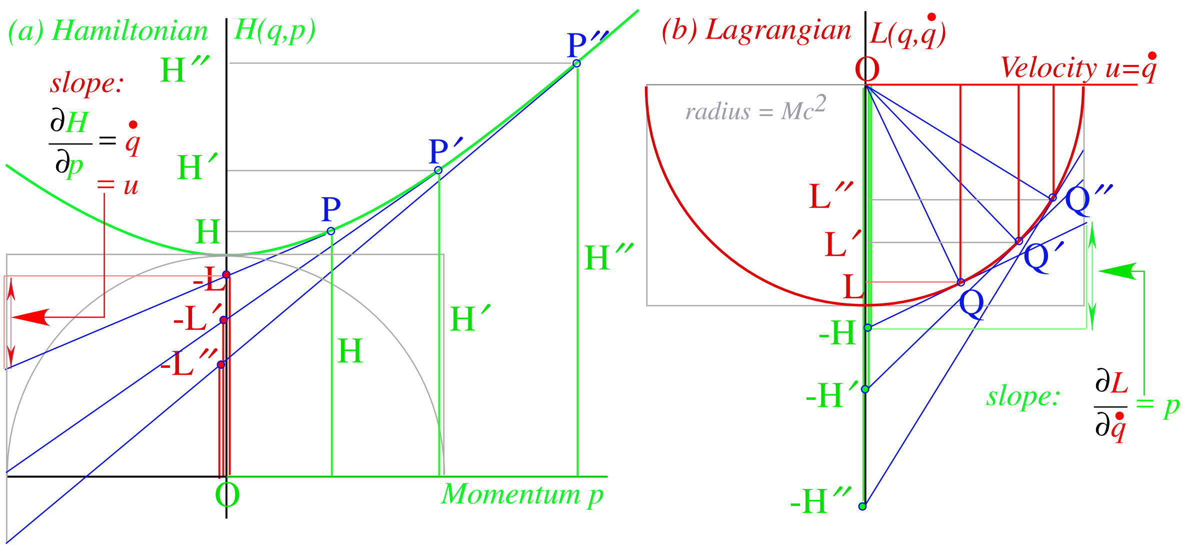


Fig. 6 Geometry of contact transformation between relativistic (a) Hamiltonian (b) Lagrangian

Action integral $S=\int Ldt$ is to be *minimized*. Feynman's interpretation[25] of S minimization is depicted in Fig. 7. A mass flies so that its "clock" τ is *maximized*. Proper frequency $\mu = Mc^2 / \hbar$ is constant for fixed rest mass, and so minimizing $-\mu\tau$ means maximizing $+\tau$. Huygen's wave interference then demands stationary and extreme phase, that is, the fastest possible clock.

Thus a Newtonian clockwork-world appears to be the perennial cosmic gambling-house winner in a kind of wave dynamical lottery in an underlying wave fabric. Einstein's God may not play dice. Yet, some persistently wavelike entity seems to be gaming *24&7* down in the cellar!

It is ironic that Evenson and other metrologists made the greatest advances of precison in human history, not with metal bars or ironclad classical mechanics, but with light waves, some of most ethereal and dicey stuff in the universe. This helps us to see that classical matter or particle mechanics is more simply and elegantly described through its relation to light waves with their manifestly annoying properties of relativity, interference, and quantization. Newton complained that light seemed to "have fits." Perhaps, it is trying to tell us something.

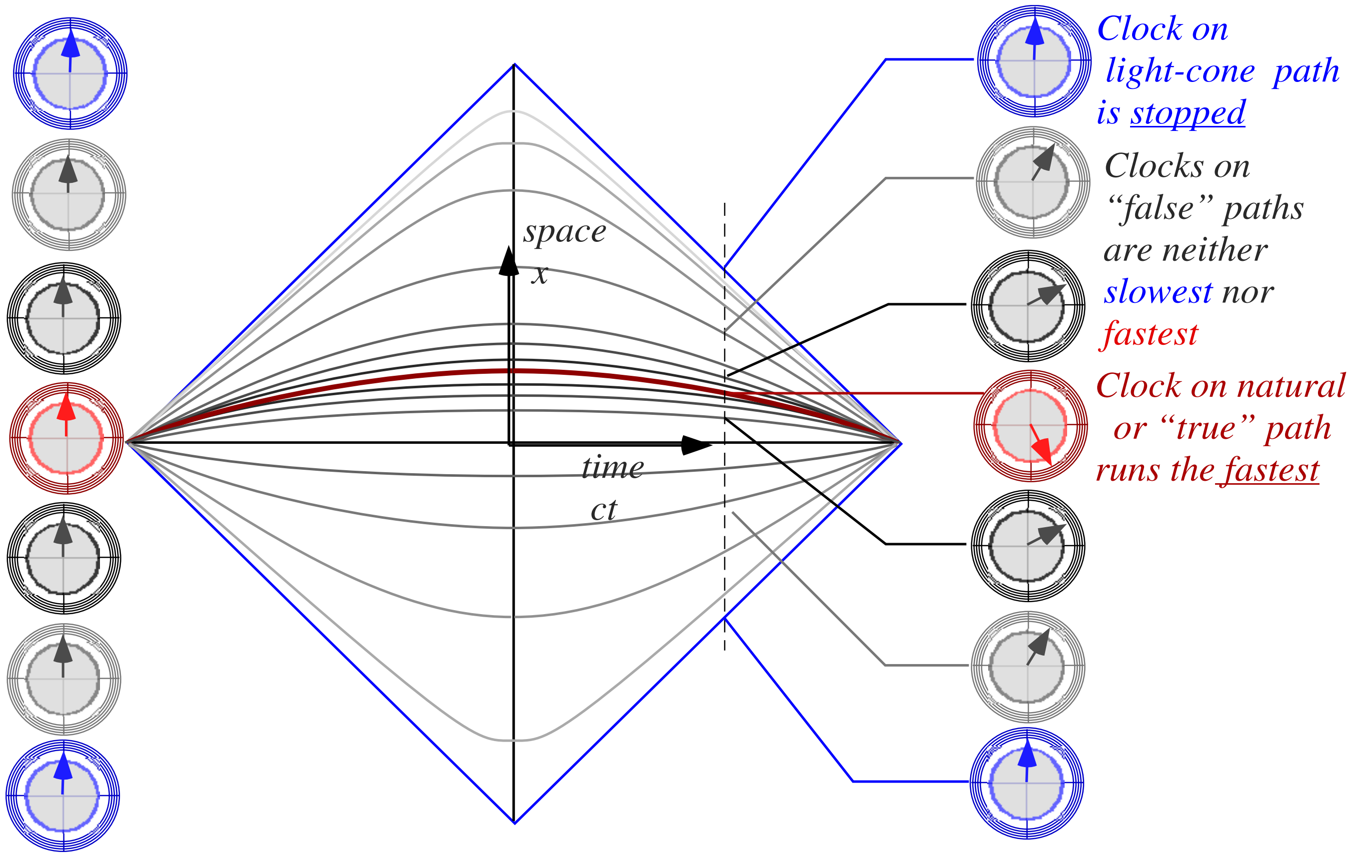


Fig. 7 "True" paths carry extreme phase and fastest clocks. Light-cone has only stopped clocks.

Matter-light waves and Compton effects

Relations (15-23) suggest connections between matter waves of mass M and light quanta of phase frequency ω_p based upon an Einstein-Planck relation $\hbar\omega_p = Mc^2$. Quantum optics can model mass kinematics in the spirit of Einstein *gedanken* experiments that pioneered idealized models of photo-kinetics known later as Compton effects.[26] Mass-dispersion curves in Fig. 3 also model an ideal "ultra-light" laser cavity, a kind of *gedanken* device with 100 years of hindsight.

The lowest cavity mode is cosine-standing-wave mode-*c1* in (1) and Fig. 1c. Its lowest quantum state is a 1-photon state $|N_{c1}=1\rangle = (\mathbf{a}_{c1}^\dagger)|0\rangle$ created by a boson operator combination $\mathbf{a}_c^\dagger = \frac{1}{\sqrt{2}}\mathbf{a}_1^\dagger + \frac{1}{\sqrt{2}}\mathbf{a}_3^\dagger$ of 1st base and 3rd base plane waves. State $|N_{c1}=1\rangle$ has proper frequency ϖ and is seen in any frame to have its phase **K**-vector on a hyperbola-*(N=1)* in Fig. 3c and Fig. 8.

Next is hyperbola-*(N=2)* for a 2-photon state $|N_c=2\rangle = (\mathbf{a}_c^\dagger)^2|0\rangle$ created by two $\mathbf{a}_c^\dagger$ factors $(\mathbf{a}_c^\dagger)^2 = \frac{1}{2}(\mathbf{a}_1^\dagger)^2 + \mathbf{a}_1^\dagger\mathbf{a}_3^\dagger + \frac{1}{2}(\mathbf{a}_3^\dagger)^2$ with each $\mathbf{a}_c^\dagger$ contributing its phase factor $(e^{-i\varpi t})$ to give a total proper frequency 2ϖ. 2-particle states are products $|n_1\rangle|n_3\rangle$ or $\mathbf{a}_1^\dagger\mathbf{a}_3^\dagger$ of individuals so probability P_{13} for state *1 and 3* is a product $P_1 P_3$ of separate probabilities. A state of *two* ϖ photons models the kinematics for a mass equivalent $M_{rest} = 2\hbar\varpi/c^2$. This is related by symmetry to Dirac's theory[27] of 1925 and to Anderson's electron-positron creation $(2\gamma \rightarrow e + \bar{e})$ experiments of 1939.[28]

Symmetry and conservation (Sort of)

Elegance of wave-based derivation owes a lot to symmetry principles. Lorentz symmetry requires invariance to translation $\mathbf{T}(\delta,\tau)$ in space and time. $\mathbf{T}$ has plane wave eigenfunctions $\psi_{k,\omega}$ with eigenvalues $e^{i(k\cdot\delta-\omega\cdot\tau)}$. Such a simple result is powerful if product states $\Psi_{K,\Omega}=\psi_{k_1,\omega_1}\psi_{k_2,\omega_2}$, such as Dirac's $e+\bar{e}$ pairs, also have $\mathbf{T}(\delta,\tau)$-eigenvalues $e^{i(K\cdot\delta-\Omega\cdot\tau)}$. Then overlap $\langle\Psi'_{K',\Omega'}|\Psi_{K,\Omega}\rangle$ is zero unless $K'=k'_1+k'_2=K$ and $\Omega'=\omega'_1+\omega'_2=\Omega$. This implies $E=\hbar\Omega$ and $P=\hbar K$ are *conserved*.

$$\begin{aligned}\langle\Psi'_{K',\Omega'}|\Psi_{K,\Omega}\rangle &= \langle\Psi'_{K',\Omega'}|\mathbf{T}^\dagger(\delta,\tau)\mathbf{T}(\delta,\tau)|\Psi_{K,\Omega}\rangle \\ &= \langle\Psi'_{K',\Omega'}|e^{-i(K'\cdot\delta-\Omega'\cdot\tau)}e^{i(K\cdot\delta-\Omega\cdot\tau)}|\Psi_{K,\Omega}\rangle = 0 \text{ unless: } K'=K \text{ and: } \Omega'=\Omega\end{aligned}$$

Energy-momentum conservation is a wave *theorem* trumping 300-year old *axioms.* Real-world γ or *M*-waves put limits $\Delta\mathbf{x}$ on $\mathbf{T}$-symmetry so tiny **K**-uncertainty $\Delta K\cong 1/\Delta x$ may *dis*prove Newton "laws" ever so slightly. Nevertheless, to the extent it approximates $\mathbf{K}_2$ of a mass *M*, a sum $\mathbf{K}_1+\mathbf{K}_3$ of photon bases *must* share the hyperbola, symmetry and kinematics of that mass *M*.

Symmetry is to science what religion is to politics. Both are deep and grand in principle but roundly flaunted in practice. Also, both get success and failure by overlooking details and embarrassing questions like, "What "cavity" traps Dirac's $\frac{1}{2}MeV$ γ- pairs into stable $e+\bar{e}$ pairs?"

Answering Feynman's father (Sort of)

Feynman recalls being unable to answer his father's question, "Where is a photon before an atom emits it?" This was after his father had paid for a pricey MIT education.[29] Let's try out an answer that makes Compton calculations a geometric delight. It is based on a model in Fig. 8.

Using the baseball diamonds in Fig. 1 or 3, we say atoms *are* photon pairs or at least have the *symmetry* of 2-photon states represented by 2nd base $\mathbf{K}_2$. In the lower half of Fig. 8a, a 1st base $\mathbf{K}_1=(\varpi,\varpi)$ and 3rd base $\mathbf{K}_3=(-\varpi,\varpi)$ sum to an atom's 2nd base $\mathbf{K}_2=(0,2\varpi)$ on a hyperbola of mass $M_Q=2\hbar\varpi/c^2$ at Q. (The pitcher's mound P represents a 1-photon expectation value $\mathbf{K}_p=\frac{1}{2}\mathbf{K}_2$.)

In Fig. 8a emitted photon $\omega_{QP'}$ is "cut" from 3rd base so $\omega_3=\varpi$ shrinks by what we call[30] a *father-Feynman factor* $ff(\omega'_3=ff\varpi)$. 1st base stays at its old value $(\omega'_1=\varpi=\omega_1)$ and 2nd base moves from point Q on its initial hyperbola 2ϖ to P' on hyperbola $2\varpi'$. This new proper frequency is a geometric mean $2\varpi'=2\sqrt{\omega'_3\omega'_1}=2\sqrt{ff}\varpi$ or *Feynman*[31]*redshift* $f\equiv\sqrt{ff}$ of ϖ' to its new 3rd base value $(\omega'_3=f\varpi'=ff\varpi)$. An inverse-shift of ϖ' is its old 1st base value $(\omega'_1=f^{-1}\varpi'=f^{-1}f\varpi=\omega_1)$. In Fig. 8a old 1st base and new 3rd base span a diamond of rapidity ρ like the one in Fig. 2b where $e^{-\rho}=\frac{1}{2}$. That redshift $\varpi'/\varpi=f$ is the final-to-initial rest mass ratio $(f=\frac{1}{2})$ used in Fig. 8a-c.

$$e^{-\rho}=f\equiv\sqrt{ff}=\varpi'/\varpi=M_P/M_Q \tag{35}$$

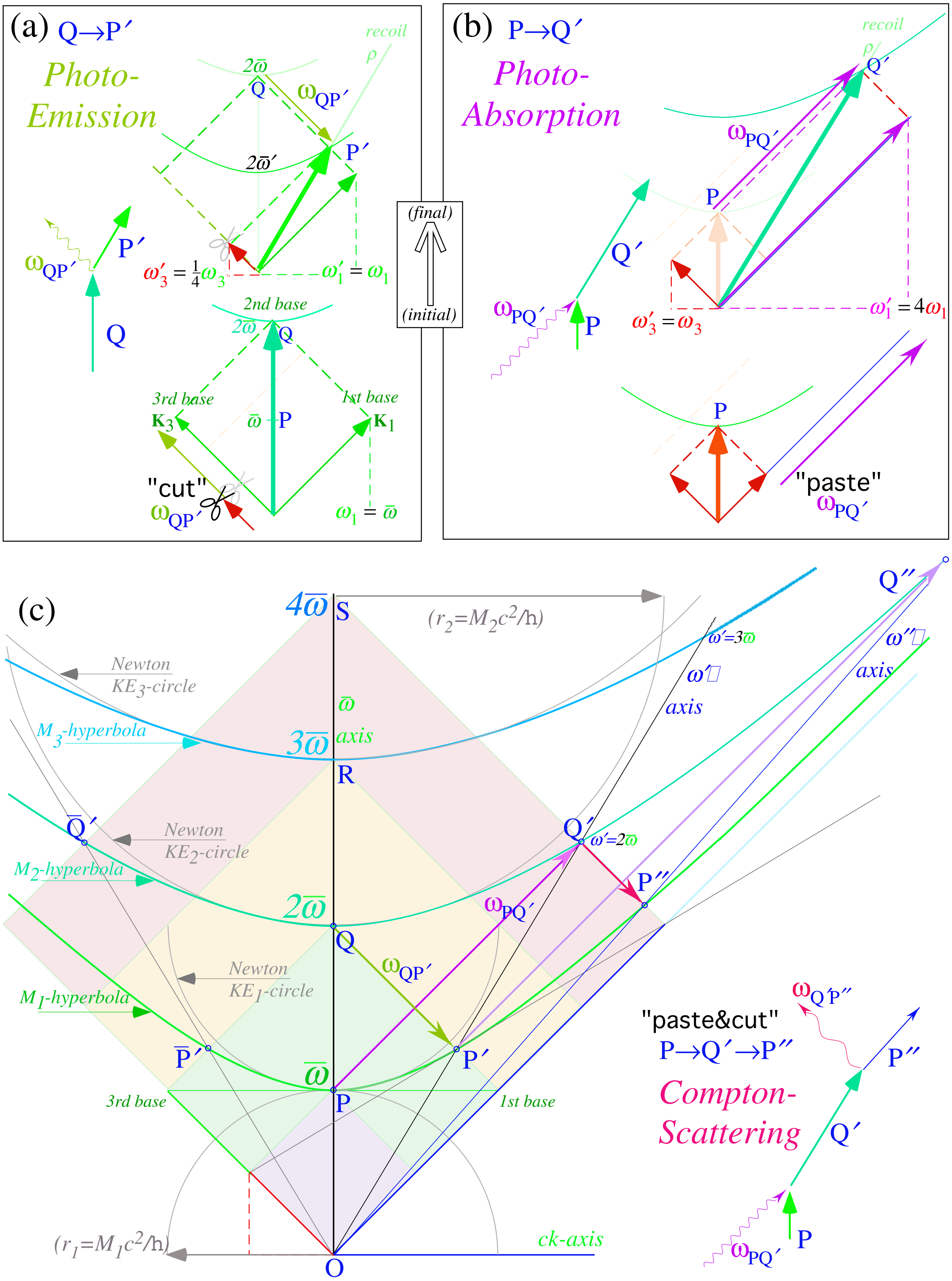


Fig. 8 Optical cavity model of (a) Emission, (b) Absorption, and (c) Compton scattering

The father-Feynman factor $(ff = \frac{1}{4})$, chosen in Fig. 8, cuts a fraction $(1 - ff = \frac{3}{4})$ off the 3rd base photon $\omega_3 = 2\varpi/2$. Emitting $\omega_{QP'} = \frac{3}{4}\varpi$ reduces mass M_2 by Feynman factor $f = \sqrt{ff} = \frac{1}{2}$ to M_1, doubles its Doppler factor $(f^{-1} = 2 = e^{\rho})$ and recoils at velocity $u = \frac{3}{5}c$. (Recall Fig. 2.) Mass M_1 gets the same boost in a "paste" by absorping $\omega_{PQ'} = \frac{3}{2}\varpi$ as it jumps from P to Q′ in Fig. 8b, opposite to the $\omega_{QP'}$ "cut" that jumps Q to P′ in Fig. 8a. The ω'-axis P′Q′ has recoil $\rho = \ln 2$ in Fig. 8c.

Another recoil $\rho = \ln 2$ results by emission $\omega_{Q'P''}$ in the final step of a Compton "paste-and-cut" process having the Feynman diagram in Fig. 8c (lower right). The Feynman segments, drawn to scale, form a bent "kite" OPQ′P′O transformed from a symmetric kite O$\bar{\text{P}}'$QP′O by the same Lorentz boost by $\rho = \ln 2$ that transforms the main kite OQ- axis to its O$\bar{\text{P}}'$ or OP′ wings.

Thus both "paste-and-cut" $(P_{\rightarrow}Q'_{\rightarrow}P'')$ and inverse "cut-and-paste" $(Q_{\rightarrow}P'_{\rightarrow}Q'')$ processes entail total recoil $2\rho = \ln 2^2$ from the lab ϖ axis to an ω'' axis. The latter absorbs photon $\omega_{P'Q''} = 3\varpi$ that moves it from rapidity ρ on hyperbola-ϖ to rapidity 2ρ on hyperbola 2ϖ, that is, a fast $(\frac{u}{c} = \frac{3}{5})$ mass M_1 to a faster $(\frac{u}{c} = \frac{15}{17})$ and heavier mass $M_2 = 2M_1$ at point Q″ in the upper right corner of Fig. 8c.

An atom's transitions are tiny compared to its rest mass and not harmonically spaced like the optical-cavity model used here. (Our *2:1*- rest mass drop shows geometry better than a more realistic ratio $10^{10}:10^{10}-1$.) Any hyperbolic energy level ratio $m:n$, integral or not, is possible.

Compton-Doppler staircases

If a lab-fixed atom drops from level $n\varpi$ to $m\varpi$ its recoil shift is $f_{nm} = e^{-\rho_{nm}} = \frac{m}{n}$ by (35). Its emitted frequency ω_{nm} is the altitude of a kite triangle, like $\bar{\text{P}}'$QP′ in Fig. 8c, given as follows.

$$\omega_{nm} = (1 - f_{nm}{}^2)\frac{n\varpi}{2} = \frac{n^2 - m^2}{2n}\varpi = n\varpi e^{-\rho_{nm}} \sinh\rho_{nm} \qquad (36)$$

The example in Fig. 8a has $\omega_{QP'} = \frac{3}{4}\varpi = \omega_{2,1}$. Doppler shifts of $\omega_{2,1}$ by $f_{2,1} = \frac{1}{2}$ form a geometric series $(\cdots, \frac{3}{32}, \frac{3}{16}, \frac{3}{8}, \frac{3}{4}, \frac{3}{2}, 3, 6, 12, \cdots)\varpi$ of possible spectra in a Compton staircase $PQ'P''Q''\ldots$ of *(2:1)*-levels 2ϖ and 1ϖ in Fig. 8c. For an integral *(m:n)* level ratio, each dilation factor γ_{nm}, recoil β_{nm}, or ratio ω_{nm}/ϖ is rational. The Pythagorean sum $1 = \gamma_{nm}^{-2} + \beta_{nm}^2$ has a rational triangle, *e.g.*, $1 = \frac{3^2}{5^2} + \frac{4^2}{5^2}$.

$$\beta_{nm} = \frac{u_{nm}}{c} = \tanh\rho_{nm} = \frac{n^2 - m^2}{n^2 + m^2}, \quad \gamma_{nm} = \cosh\rho_{nm} = \frac{n^2 + m^2}{2mn}, \quad \sinh\rho_{nm} = \frac{n^2 - m^2}{2mn}$$

Hyperbolic recoil KE_m makes emitted ω_{nm} less than $|n - m|\varpi$ by a factor $(n+m)/2n$. Absorbed ω_{mn} is more by $(n+m)/2m$. Low-u Newtonian $KE_n \cong M_n u^2/2$ are circles of radius $M_n c^2$. Mass recoil always gets a cut. Like money-changing tourists, light waves get nicked coming and going.

Since $\omega_{mn} = \omega^{IN}$ is greater than ω_{nm} by a factor $f_{mn} = \frac{n}{m}$, a Compton ω^{OUT} due to ω^{IN} is less than ω_{nm} by the inverse factor $f_{mn}^{-1} = f_{nm} = \frac{m}{n}$ and less than ω^{IN} by its square $ff = f_{nm}^2 = (\frac{m}{n})^2$.

$$\omega^{IN} = \omega_{mn} = \frac{n}{m}\,\omega_{nm}\ , \qquad \omega^{OUT} = \frac{m}{n}\,\omega_{nm} = (\frac{m}{n})^2\,\omega^{IN} \qquad (37)$$

Compton processes are 2-photon jumps off "virtual" intermediate $\omega_n = n\varpi$ hyperbolas that begin *and* end on *one* hyperbola $\omega_m = m\varpi$. If integers *m*, *n* in the cavity model are real values of atomic mass-energy then (37) applies to levels in real atoms and similarly for 1-photon relations (36).

Inverse frequencies $\omega^{-1} = (kc)^{-1} = \lambda(2\pi c)^{-1} \equiv \lambda\!\!\!^{-} / c$ give Compton's wavelength sum rule.

$$(\omega^{OUT})^{-1} = (\omega^{IN})^{-1} + 2\hbar(m\varpi)^{-1} \text{ , or: } \lambda\!\!\!^{-}{}^{OUT} = \lambda\!\!\!^{-}{}^{IN} + 2\lambda\!\!\!^{-}_C \text{ where: } \lambda\!\!\!^{-}_C = \frac{\hbar c}{m\varpi} = \frac{\hbar}{M_m c}. \quad (38)$$

Compton radius $\lambda\!\!\!^{-}_C \equiv \lambda_C / 2\pi$ is a minimum size for a cavity of mass M_m. Input $\lambda\!\!\!^{-}{}^{IN}$ picks up $2\lambda\!\!\!^{-}_C$ as $\lambda\!\!\!^{-}{}^{OUT}$ bounces off an "M_m box" of size $\lambda\!\!\!^{-}_C$ that is a function of terminal *m* but *not* of intermediate *n*. Larger mass M_m fits in a *smaller* box and recoils less while giving a more elastic photon bounce.

A geometric ϖf^p -series $\varpi(\ldots f^{-2}, f^{-1}, 1, f^1, f^2 \ldots)$ of *levels* has $f^p \left| f^2 - 1 \right| \frac{\varpi}{2}$ -series of *transitions* that form Compton "nets" such as the $(f = 2)$-net in Fig. 9a or a finer $(f = \sqrt{2})$-net in Fig. 9b.

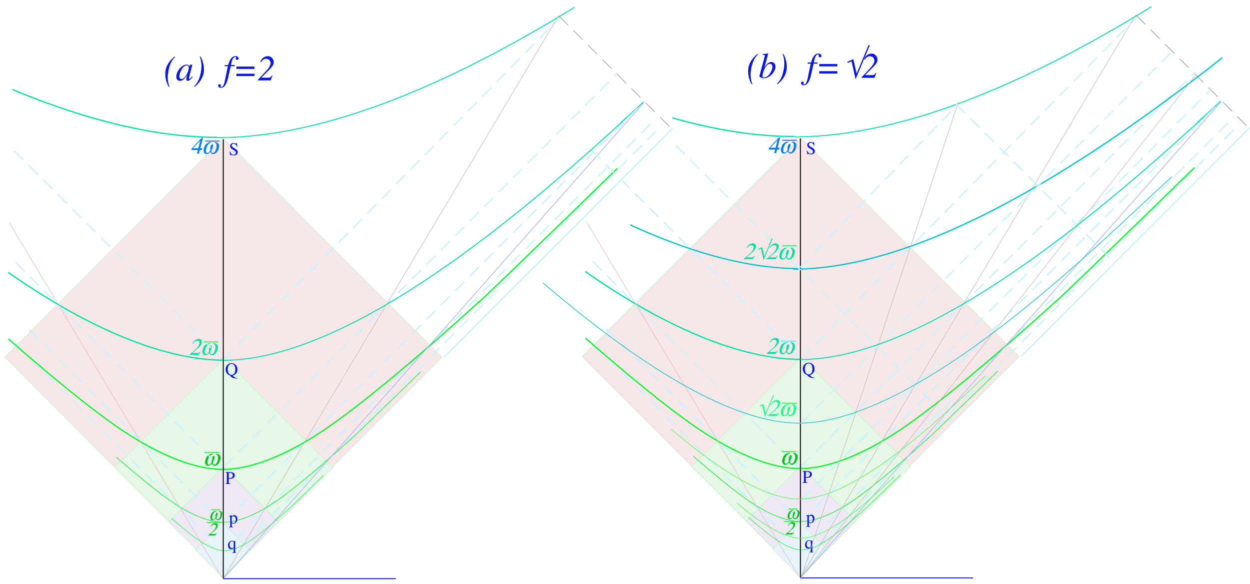


Fig. 9 Compton nets are congruent Compton staircases of transitions. (a) f=2:1 (b) f= $\sqrt{2}$

Colorful rides on Einstein elevators

A spacetime version of Compton nets are curved coordinates for accelerated Einstein elevators and helps to visualize equivalence principles for general relativity.[32] Plots in Fig. 10-11 show waves from chirping tunable lasers forming colorful renditions of hyper-net coordinates.

A previous Fig. 2c plotted an atom *(x′,ct′)*-view of it running head-on at rapidity ρ into a green ϖ -beam that is blue ($\varpi\, e^{+\rho}$) shifted while the receding laser appears red ($\varpi\, e^{-\rho}$) shifted. The laser *(x, ct)*-grid then appears as a ρ-tipped Minkowski grid. If instead the lasers had been tuned to frequencies $\varpi\, e^{-\rho}$ and $\varpi\, e^{+\rho}$, respectively, the (u=$c$tanh ρ)-moving atom could see both beams to be green ϖ –light waves interfering to make a square *(ρ=0)* Cartesian *(x, ct)*-grid like Fig. 1c.

Varying the tuning parameter ρ of the lasers changes local grid rapidity ρ at the beams' spacetime intersection as sketched in Fig. 10a-b. This produces a curved spacetime coordinate system of paths with rapidity changing just so *both* beams end up the *same* color on a given path.

Each trajectory plotted in Fig. 11 has its own constant proper acceleration g and local color ϖ. A mass M following such a $x(t)$-path also follows its M-hyperbola in Fig. 9. The lasers each send waves that meet at each trajectory point $x(t)$ and paint a local interference grid of varying rapidity ρ on a trajectory $x(t)$ of varying velocity $u(t)$ given by (6a) and sketched in Fig. 10a.

$$u = \frac{dx}{dt} = c\tanh\rho \tag{39}$$

Setting $x'=0$ and $t'=\tau$ in (11) relates proper time interval $d\tau$ to lab dt . This gives $x(t)$ by τ-integrals.

$$\frac{dt}{d\tau} = \cosh\rho \quad (40a) \qquad \frac{dx}{d\tau} = \frac{dx}{dt}\frac{dt}{d\tau} = c\tanh\rho\cosh\rho = c\sinh\rho \quad (40b)$$

$$ct = c\int\cosh\rho \;\; d\tau \quad (40c) \qquad x = c\int\sinh\rho \;\; d\tau \quad (40d)$$

Path $x(t)$ depends on $\rho(\tau)$ variation in proper τ. Linear rate $u{\sim}g\tau$ or $\rho{=}g\tau/c$ gives a hyperbolic path in Fig. 10b of fixed proper acceleration g and a family of concentric paths of different g in Fig. 11.

$$ct = c\int\cosh\left(\frac{g\tau}{c}\right)\, d\tau = \frac{c^2}{g}\sinh\left(\frac{g\tau}{c}\right) \quad (41a) \qquad x = c\int\sinh\left(\frac{g\tau}{c}\right)\, d\tau = \frac{c^2}{g}\cosh\left(\frac{g\tau}{c}\right) \quad (41b)$$

Paths closer to the left hand blue-chirping laser have a higher g than flatter ones nearer the red-chirping right hand source. ρ-skewed baseball diamonds of PW and CW paths in lower Fig. 11 are spaced geometrically along the x-axis of a spaceship at a moment when its rapidity is $\rho{=}0.2$.

Geometric $e^{\pm\rho}$-variation (41) of wave and coordinate spacing is due to a left-hand laser's right-moving wave of frequency $\omega_{\rightarrow}{=}\omega_0 e^{+\rho}$ on light cone $x_{\rightarrow}{=}x{-}ct{=}x_0 e^{-\rho}$ and a right-hand laser's left-moving wave of frequency $\omega_{\leftarrow}{=}\omega_0 e^{-\rho}$ on light cone $x_{\leftarrow}{=}x{+}ct{=}x_0 e^{+\rho}$. Wave interference does the rest.

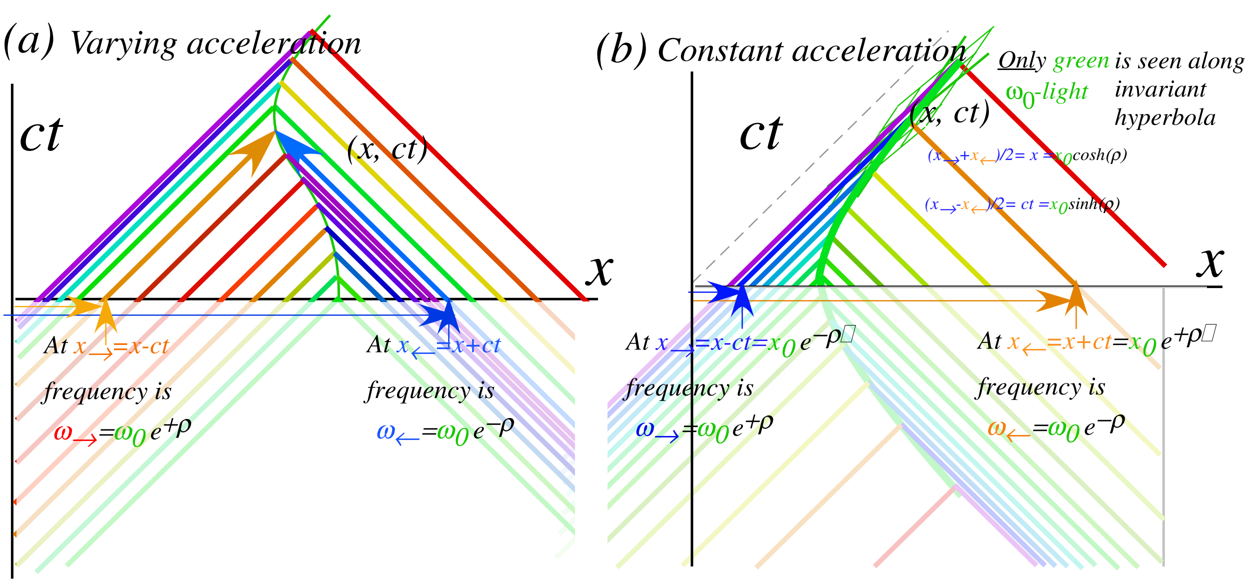


Fig.10 Optical wave frames by red-and-blue-chirped lasers (a)Varying (b)Constant acceleration

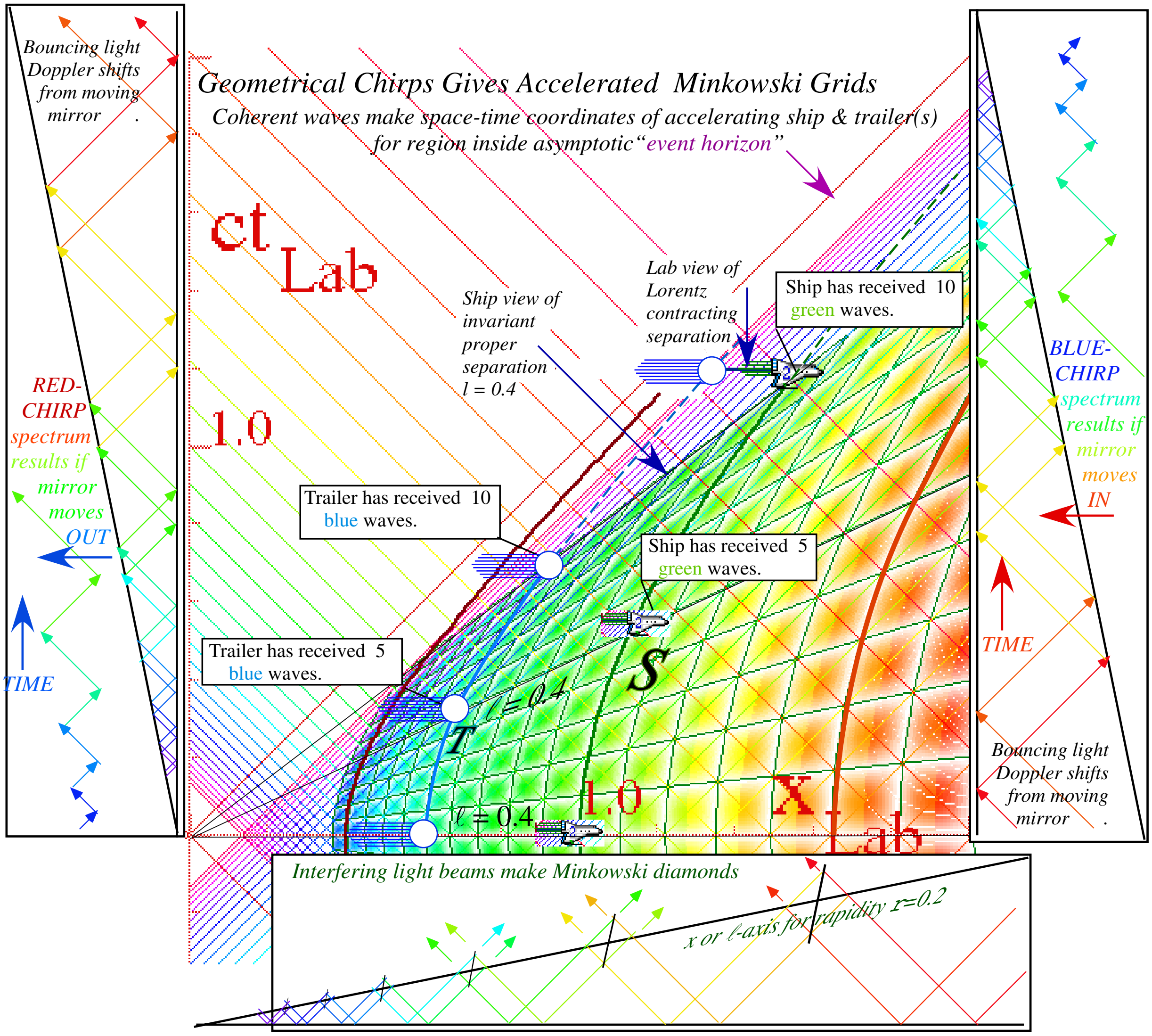


Fig. 11 Accelerated reference frames and their trajectories painted by chirped coherent light

Initial $(\rho=0)$ position of hyperbola ω_0 is $\ell_0=x_0=c^2/g_0$. Each hyperbola has different but fixed location ℓ, color ω, and gravity g that, by (41), are proper invariants of each path.

$$x^2-(ct)^2 = \ell^2 , \quad \text{where: } \ell=c^2/g \tag{42}$$

Frequency ω and acceleration g vary inversely with the path's proper location ℓ relative to origin.

$$\omega\, \ell = \omega\, c^2/g = \omega_0\, c^2/g_0 = const. \tag{43}$$

Rapidity $\rho=g\tau/c$ in (41) has proper time be a product of hyperbolic radius ℓ in (42) and "angle" ρ.

$$c\tau = \rho\, c^2/g = \ell\, \rho \tag{44}$$

This is analogous to a familiar circular arc length formula $s = r\,\phi$. Both have a singular center.

The less familiar hyperbolic center $(x,ct)=(0,0)$ here begins an elementary *event horizon*. The blue-chirp laser would need infinite frequency $\omega_0 e^{+\rho}$ at origin where $ct=e^{-\rho}$ goes to zero, so it gives up *before t=0*. After $t=0$, light from the laser to any path S or T given by (41) never arrives. Fig. 11 shows paths of a spaceship S and a "trailer" T trailing by invariant length $\ell_{ST}=\ell(S)-\ell(T)$ on an x-axis of rapidity ρ through origin $(x,ct)=(0,0)$. S and T always have the *same velocity* (39) relative to the lab, maintain proper interval ℓ_{ST} , but trailer *T feels greater g*. Lower parts of a rigid rod accelerate more to get a correct lab Lorentz length-contraction indicated at the top of Fig. 11.

In a Newtonian paradigm, asymmetric acceleration seems paradoxical, but if waves make a coordinate frame, asymmetry is a consequence of the DeBroglie relation (16) between k-vector and momentum. Accelerating frames means shortening wavelength and this crowds waves.

Wave properties also manifest the accelerated frames' upstairs-downstairs disparity in proper time τ ("later" upstairs by (44)) and shift in frequency ω (lower or "red shifted" upstairs by (43)). Along nodal (white) lines that are the ship-trailer x-axis for a momentary rapidity ρ, wave phase is seen to be some constant $k\ell-\omega\tau=N\pi/2$. The Einstein equivalence of gravity to an accelerated elevator is manifested by both a gravitational red shift and an increase of clock rates in the upstairs regions of a field.

A quantized version of Fig. 11 would be an atom with a transition at ω_I, undergoing a sequential *resonant* Compton scattering of exponentially chirped photons ω_I, $e^{\pm\rho}\omega_I$, $e^{\pm 2\rho}\omega_I$, $e^{\pm 3\rho}\omega_I$,… between the *same* pair of hyperbolas in Fig. 9. The atom sees the *same* color and feels the *same* recoil rapidity at each step in the quantum version of constant acceleration.

Constant velocity gives constant acceleration

This leads one to ask if chirped light might be used for atomic or molecular acceleration. Logarithmic dependence $\rho=\ln b$ of rapidity on the shift factor b favors ultra-precise *low* energy acceleration, perhaps for nanotechnology. (High power would require extreme bandwidth.)

The flip symmetry between two sides of a lightcone suggests optical cavities with a geometric chirp. If you flip the diamond sequence in lower Fig. 11 across the light cone to the sides of Fig. 11 you get spacetime light paths bouncing between mirrors moving relative to each other. If mirrors close, trapped light blue-chirps exponentially as on the right side in Fig. 11. It red-chirps if the two mirrors separate as they do on the left side of Fig. 11. Together, a desired $e^{\pm n\rho}$ spectrum is made simply by translating one etalon cavity at constant velocity relative to another stationary cavity that is enclosed by the translating one.

In this way, light generated by mirrors of constant *velocity* provides the spectrum needed to make an interference net of constant *acceleration*. Remarkably, this is true even at relativistic speeds, but at far lesser speeds, it is possible that coherernt acceleration like Fig. 11 (but slower) might be done with great precision. Length metrology must eventually use waves, too!

Wave geometry ought to make us more skeptical of the coordinate boxes and manifolds that we carry about in our minds. A common image is the Newton-Descartes empty-box at some absolute time existing whether or not it contains any "particles." Some learn to picture spacetime coordinates as a giant metal frame of clocks like *Fig. 9* in Taylor and Wheeler's[33] relativity text. That figure is more like a parody of common views of spacetime manifolds that remain with us to this day. Such a monstrosity of a framework is decidedly nonexistent and non-operational.

In contrast, a wave frame like Fig. 1, 2, and 11 is *physical* metrological coordinate system whose geometry and logic is both revealing and real. The things being coordinated (waves) come with their own coordinates and theorems built in. Einstein general theory of relativity trumped Newton's box by showing how it is curved by any energy or mass it holds. Quantum theory is now beginning to show that, perhaps, the box and its contents could be one and the same thing.

Pair creation and quantum frames. What's ahead?

Dirac, before others, realized that per-spacetime has the symmetry of spacetime. Past and future (time-reversal) symmetry demands positive and negative frequency and asks us to play on back-to-back baseball-diamonds with *four* hyperbolas. Examples of pair-creation are sketched in Fig. 12 as seen from two different reference frames. It is a strange sort of Compton process.

The Feynman graph of Compton scattering in Fig. 8c is turned on its side in Fig. 12 so it may start and end on different branches of the m-hyperbola corresponding to mass $\pm m$. Two photons, whose energy sum equals the energy gap $2mc^2$, are shown bouncing off intermediate hyperbolas in Fig. 12 that are conjugate hyperbolas defining group wavevectors $\mathbf{K_g}$ in Fig. 1 or 2, and belong to *instanton* or *tachyon* waves of imaginary frequency $\pm i\mu$ and huge damping $e^{-mc^2\tau/\hbar}$.

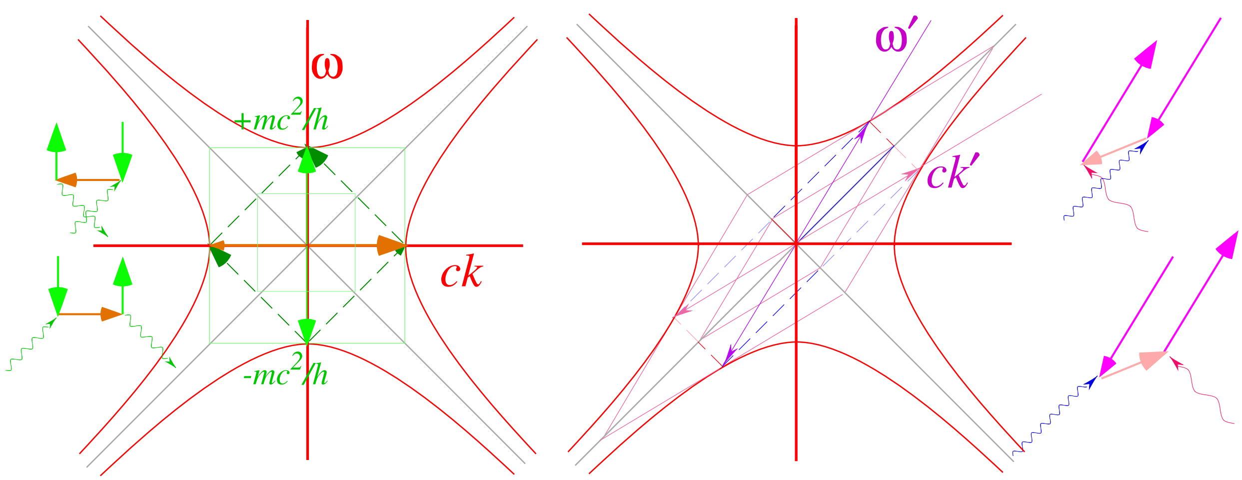


Fig. 12 Dirac matter-antimatter dispersion relations and pair-creation-destruction processes.

Dirac's is the first quantum theory to fully incorporate relativity. Introducing dual anti-worlds, in which all three mass definitions (21-23) have negative values, raises serious questions about their physical meaning. Analogies between the $(2\gamma \rightarrow e + \bar{e})$ process in Fig. 12 and exciton formation in band theory of solids, shed some light on the physics. However, the exciton process is a straight-up 1-photon process whose momentum is tiny compared to the energy jump, and it lacks the world-anti-world symmetry of the Dirac exciton in which both the electron and an anti-electron have the same group velocity but opposite momentum. The Dirac model's duality of reversed energy (frequency), momentum (k-vector), space, and time is quite extraordinary.

A number of implications of Dirac's theory have been mostly ignored. Physicists are unwilling to abandon vestigial concepts associated with absolute classical frames or "boxes." However, quantum frames are like all things quantum mechanical and have an intrinsic *relativity* associated with their wavelike interference. Quantum frames, as they are used in molecular and nuclear physics, are known to have "inside" or body-relative parts in addition to the more well known "outside" or laboratory-relative parts. This inside-and-out duality is a deep quantum mechanical result arising first in the theory of quantum rotors by Casimir, but it also underlies Lorentz-Poincare symmetry theory that includes rotating frames as well as translating ones.

Indeed, the full quantum theory of angular momentum has a built-in duality that is as fundamental as the left-and-right or bra-and-ket duality of the conjugate parts of Dirac's elegant quantum notation. The Wigner $D^J_{m,n}$ –functions are quantum rotor wavefunctions $D^{J*}_{m,n}(\alpha\beta\gamma)$ that have their standard laboratory *m*-quantum numbers on the left and their "internal" or body *n*-quantum numbers on the right. Their *J*-multiplicity is thus *(2J+1)-squared* and not simply the *(2J+1)* so familiar in elementary Schrodinger quantum theory.

It took many years for classical physics to fully accept Einstein's translational relativity principles. Perhaps, if the wave nature of quantum physics had already been established, the relativistic axioms would have been seen as an immediate consequence of wave interference. Indeed, these two subjects are, perhaps, *too* closely related for that to have happened.

Now quantum theory demands a more general kind of relativity involving rotation or any kind of acceleration that is a step beyond the special relativity of constant velocity. This brings up a quite controversial area first explored by Ernst Mach, the originator of *Mach's Principle*. Mach made the seemingly impossible proposal that centrifugal forces, the kind physicists assign the label "ficticious force", are somehow due to their relativity to all matter in the universe.

As silly as that sounds, a kind of quantum Mach's Principle is needed to make sense of the spectra and dynamics of quantum rotors even in the non-relativistic limit. But, a fully relativistic quantum treatment of these systems seems still a long way off in either experiment or theory. Also, it is not yet clear what if anything are the cosmological implications of such quantum wave mechanics. But it would appear that the ideas of Dirac are the ones to re-examine first.

Physics is still at a stage where large-scale phenomena use Newton-Einstein particle-in-manifold theory while small-scale phenomena employ Planck-DeBroglie-Schrodinger wave theory. However, both employ some form of space and time coordinates. In this they share an enigma whose existence is largely unquestioned. Supposed invariance to reference frame definition is taken to mean that frames don't matter.

That leaves our fundamental metrology in a dysfunctional dysphoria of an ignored spouse, indispensable, but having only marginal identity. If Evenson and Einstein have taught us anything, it is that this has to be a mistake. Frames *do* matter! The results of Dirac, Anderson, and many others has shown they *make* matter and *are* our matter.

__Appendix A: Group and phase wave frames

To distinguish group and phase velocity we need to combine two different waves that are quite unlike light. Fig. A1 shows a pair of "non-light" wave sources. The first *source-2* puts out a "red" wave of wavevector-frequency $(k_2,\omega_2)=(1,2)$ while the other *source-4* puts out a "blue" wave of wavevector-frequency $(k_4,\omega_4)=(4,4)$. One may pick four random numbers for *source-2* (k_2,ω_2) and *source-4* (k_4,ω_4) and the following description generally applies.

Wavevector-frequencies $\mathbf{K}_2=(k_2,\omega_2)$ and $\mathbf{K}_4=(k_4,\omega_4)$ give the following wave interference.

$$\Psi_{4+2}=e^{i(k_2x-\omega_2t)}+e^{i(k_4x-\omega_4t)}=2e^{i\left(\frac{k_4+k_2}{2}x-\frac{\omega_4+\omega_2}{2}t\right)}\cos\left(\frac{k_4-k_2}{2}x-\frac{\omega_4-\omega_2}{2}t\right) \tag{1}$$

Phase $e^{i()}$ and *group factor* $cos()$ has a sum $\mathbf{K}_{phase}=(\mathbf{K}_4+\mathbf{K}_2)/2$ or difference $\mathbf{K}_{group}=(\mathbf{K}_4-\mathbf{K}_2)/2$.

$$\mathbf{K}_{phase}=\frac{\mathbf{K}_4+\mathbf{K}_2}{2}=\frac{1}{2}\begin{pmatrix}\omega_4+\omega_2\\ k_4+k_2\end{pmatrix}=\frac{1}{2}\begin{pmatrix}4+1\\ 4+2\end{pmatrix}=\begin{pmatrix}2.5\\ 3.0\end{pmatrix} \tag{2a}$$

$$\mathbf{K}_{group}=\frac{\mathbf{K}_4-\mathbf{K}_2}{2}=\frac{1}{2}\begin{pmatrix}\omega_4-\omega_2\\ k_4-k_2\end{pmatrix}=\frac{1}{2}\begin{pmatrix}4-1\\ 4-2\end{pmatrix}=\begin{pmatrix}1.5\\ 1.0\end{pmatrix} \tag{2b}$$

The vectors $\mathbf{K}_2$, $\mathbf{K}_4$, $\mathbf{K}_{phase}$ and $\mathbf{K}_{group}$ are drawn in Fig. A1(b). Each slope is a *wave velocity*.

$$V_4=\frac{\omega_4}{k_4}=\frac{4}{4}=1 \tag{2c}$$
$$V_2=\frac{\omega_2}{k_2}=\frac{1}{2}=0.5 \tag{2d}$$
$$V_{phase}=\frac{\omega_4+\omega_2}{k_4+k_2}=\frac{5}{6}=0.83 \tag{2e}$$
$$V_{group}=\frac{\omega_4-\omega_2}{k_4-k_2}=\frac{3}{2}=1.5 \tag{2f}$$

The spacetime plot of wave zeros of $\mathrm{Re}\Psi$ in Fig. A1(a) shows a *group velocity* nearly twice the *phase velocity* as given above. (This is a peculiarity of Bohr waves, which these happen to be.)

____Wave lattice paths in space and time

Fig. A1 is actually a single plot that combines *spacetime (x,t)* with Fourier space or *per-spacetime* (ω,k). On wave *phase-zero paths* the real part of phase factor $e^{i(k_px-\omega_pt)}$ is zero: $k_px-\omega_pt=n_p=N_p\pi/2\ (N_p=\pm1,\pm3..)$. *Group-zero* or *nodal paths* have a zero group factor $\cos(k_gx-\omega_gt)$ if: $k_gx-\omega_gt=n_g=N_g\pi/2$. At *wave lattice points (x,t) both* factors are zero.

$$\begin{pmatrix}k_p & -\omega_p\\ k_g & -\omega_g\end{pmatrix}\begin{pmatrix}x\\ t\end{pmatrix}=\begin{pmatrix}n_p\\ n_g\end{pmatrix}\text{ where: }\mathbf{K}_{phase}=\begin{pmatrix}\omega_p\\ k_p\end{pmatrix}=\frac{1}{2}\begin{pmatrix}\omega_4+\omega_2\\ k_4+k_2\end{pmatrix},\ \mathbf{K}_{group}=\begin{pmatrix}\omega_g\\ k_g\end{pmatrix}=\frac{1}{2}\begin{pmatrix}\omega_4-\omega_2\\ k_4-k_2\end{pmatrix} \tag{3a}$$

Wave-vectors $\mathbf{K}_{phase}$ and $\mathbf{K}_{group}$ define *spacetime (x,t)* zero-paths, the white lines in Fig. A1(a).

$$\begin{pmatrix}x\\ t\end{pmatrix}=\frac{\begin{pmatrix}-\omega_g & \omega_p\\ -k_g & k_p\end{pmatrix}\begin{pmatrix}n_p\\ n_g\end{pmatrix}}{\omega_pk_g-\omega_gk_p}=\frac{-n_p\begin{pmatrix}\omega_g\\ k_g\end{pmatrix}+n_g\begin{pmatrix}\omega_p\\ k_p\end{pmatrix}}{\omega_pk_g-\omega_gk_p}=-\frac{n_p}{D}\mathbf{K}_{group}+\frac{n_g}{D}\mathbf{K}_{phase}\text{ where: }\begin{pmatrix}n_p\\ n_g\end{pmatrix}=\begin{pmatrix}N_p\\ N_g\end{pmatrix}\frac{\pi}{2} \tag{3b}$$

Phase zeros go in direction $\mathbf{K}_{phase}$ at speed V_{phase} and envelope zeros or nodes go with $\mathbf{K}_{group}$ at a speed V_{group}. Together the zeros trace out a lattice of parallelogram cells in space and time. The Lorentz-Einstein-Minkowski cells in Fig. 1 are made this way by counter-propagating laser light.

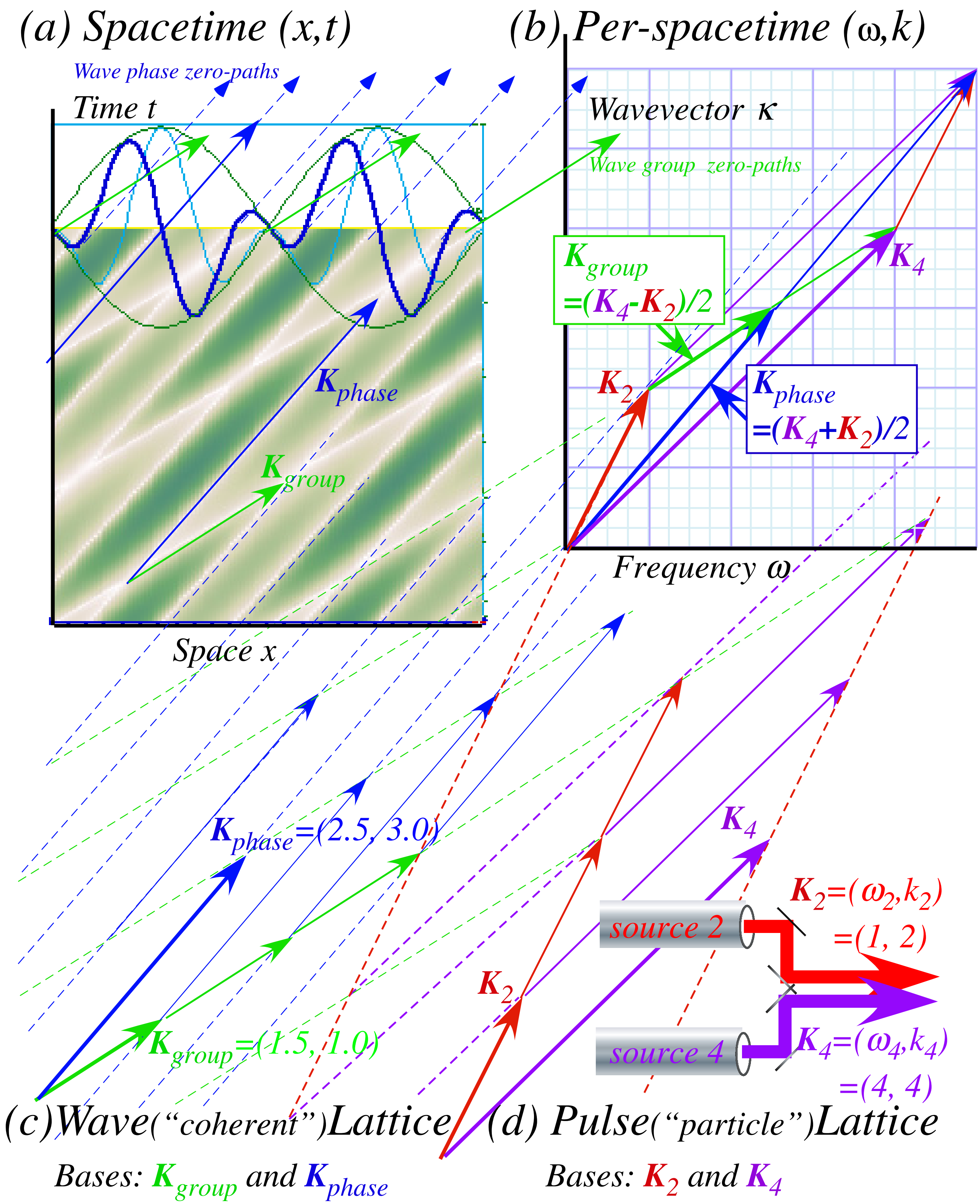


Fig. A1 Wave paths in spacetime (x,t) and Fourier per-spacetime (ω,k). (a-b) Wave zero paths along group and phase wavevectors. (c-d) Wave lattices with and without coherence.

It should be noted that the joining of a per-spacetime Fourier plot with a spacetime plot is unusual, and requires some care. First, if t is plotted versus x then (3b) requires that we plot the wavevector k versus the frequency ω instead of the other way around. (Dispersion functions $\omega(k)$ are usually plotted ω against k.) A k-*versus*-ω plot is scaled by determinant $D = \omega_p k_g - \omega_g k_p$ from (3b) so its lattice in Fig. A1(b,d) matches the x-*versus*-t wave-zero lattice in Fig. A1(a).

When that is done, the two plots may use exactly the same lattice vectors $\mathbf{K}_2$, $\mathbf{K}_4$, $\mathbf{K}_{phase}$ and $\mathbf{K}_{group}$ to define unit cells in either plot. While the $\mathbf{K}_2$ and $\mathbf{K}_4$ vectors define a primitive cell in the pulse plot of Fig. A1(d) discussed below, they also define the diagonals of the phase and group wave-zero cells spanned by $\mathbf{K}_{phase}$ and $\mathbf{K}_{group}$ in Fig. A1 (a-c). Also, the vectors $\mathbf{K}_{phase}$ and $\mathbf{K}_{group}$ define the diagonals of the primitive $\mathbf{K}_2$ and $\mathbf{K}_4$ cells as required by the vector sum relations in (2) and Fig. A1(b) and represent "corpuscular" or "pulse wave" paths.

____Particle or pulse lattice paths in space and time

A discussion of the paths of wave packet or pulses for the individual sources completes the picture. Suppose the output of the two sources could not interfere and behaved like Newton's corpuscles or particles emitted each at their assigned frequency $\omega_2=1$ or $\omega_4=4$ to go along vectors $\mathbf{K}_2$ and $\mathbf{K}_4$ at their assigned phase velocities $V_2 = 0.5$ for *source-2* particles or $V_4 = 1.0$ for *source-4* particles as given by (2c) and (2d). That is, four times as many $\mathbf{K}_4$ lattice lines as $\mathbf{K}_2$ lines cross the t-axis (or k-axis) but only twice as many $\mathbf{K}_4$ lines as $\mathbf{K}_2$ lines ($k_4/k_2=2$) are found at one time along the x-axis (or ω-axis). In other words, *source-4* goes *"patooey, patooey, patooey, patooey,…"* while *source-2* only spits half as fast, *"pa-ato-oo-oey,……………, pa-ato-oo-oey, ,……………, pa-ato-oo-oey, ,……………, pa-ato-oo-oey ,…".*

If a pulse-counter at origin $x=0$ could distinguish the "red" $\mathbf{K}_2$ from the "blue" $\mathbf{K}_4$ then it would register four times as many "blue" counts as "red" ones. All this assumes that the pulses or particles have non-dispersing Fourier components with the same phase velocity c, that is, linear dispersion $\omega=ck$, as does light. But, $\mathbf{K}_2$ and $\mathbf{K}_4$ are not on a line through origin in Fig. A1. Their dispersion is *not* linear, and complicated interference and revival effects arise from any dispersion that is not strictly linear.

____Spacetime lattices collapse for co-propagating optical waves

Fig. A2 shows the same vectors as Fig. A1 but for the combination (1) of optical or laser waves. Both V_2 for *source-2* photons and V_4 for *source-4* photons as given by (2c-d) now equal c as required by the *Colorful Relativity* axiom $\omega=ck$ that starts the wave theory of relativity. Then, the phase *and* group velocities are c by (2e-f), and scale denominator $D=\omega_p k_g - \omega_g k_p$ in (3b) is zero. So all the vectors $\mathbf{K}_2$, $\mathbf{K}_4$, $\mathbf{K}_{phase}$ and $\mathbf{K}_{group}$ collapse onto a line that holds both phase velocities and group velocities since they all have the speed of light only.

So, the optical co-propagation lattice collapses. To make a spacetime lattice with light requires *counter*-propagating waves. This leads to a simple derivation of special relativity and quantum theory and ends a 100-year-old "blind spot" in optics and modern physics.

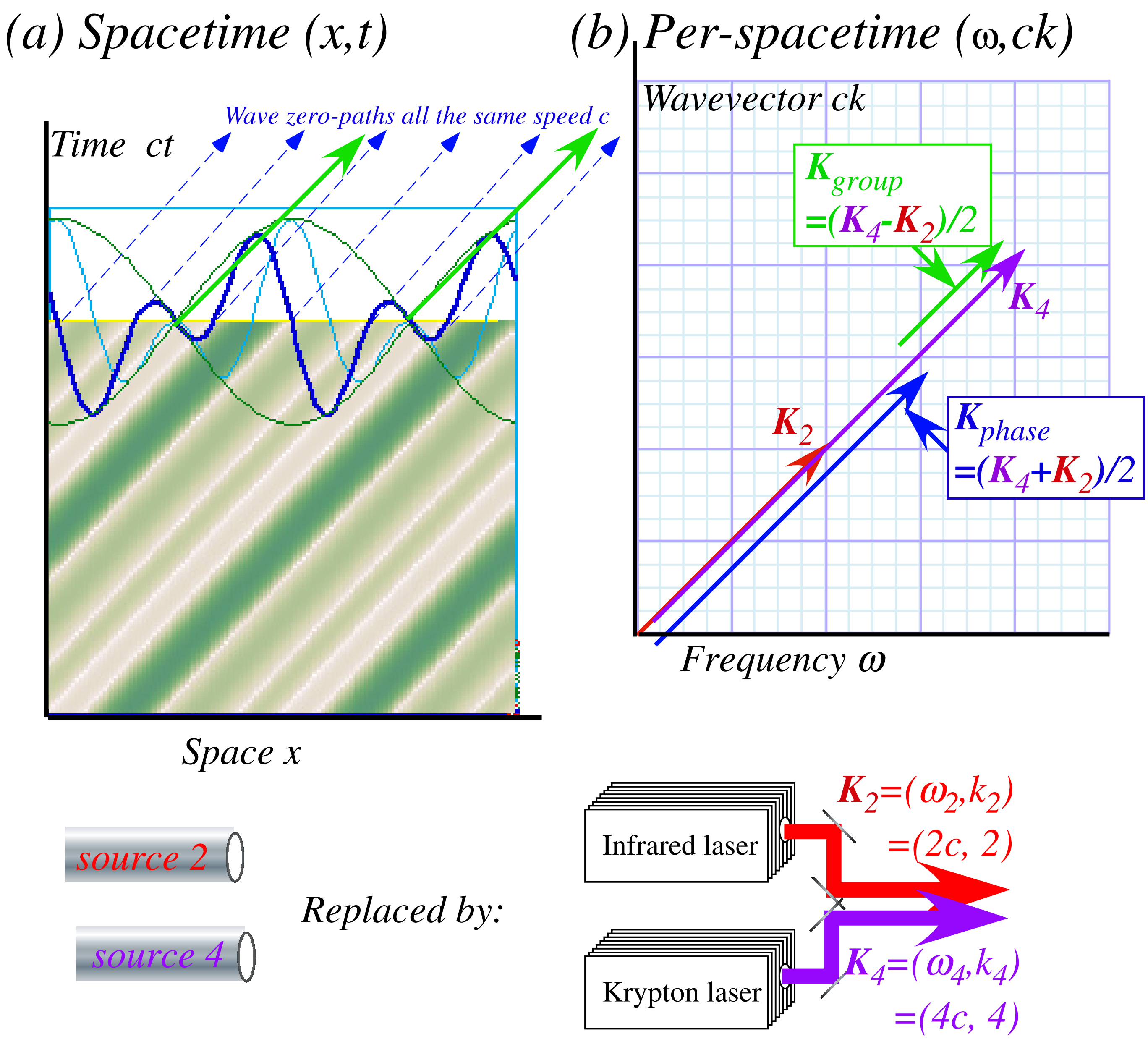


Fig. A2 Simplified wave dynamics for co-propagating optical sources.

The preceding constructions have managed to put Fourier or wave-like (per-spacetime) properties on the same page, so to speak, with Newtonian or particle-like (spacetime) ones. This is analogous to what is done in X-ray crystallographic analysis in which a real atomic position vector lattice is described using an inverse or reciprocal wavevector lattice.

References

[1] A. Cho, *String Theory Gets Real-Sort Of: The Children of the Revolution*, Science 306, 1461(2004)

[2] Dan Kleppner, Physics Today.

[3] UofA EtoE

[4] I. Newton

[5] K. Evenson

[6] Guinness

[7] P. W. Milonni (private comm.) A wonderfully tortured history of Quality factor "Q" is given by E. I. Green, American Scientist (March 1985). K. S. Johnson (Western Electric 1924) used Q because he saw it as the last letter still available at the time.

[8] William of Occam (1245-1326)

[9] Hendrik Antoon Lorentz *"Electromagnetic Phenomena in a system moving with any velocity smaller than that of light."* Koninklijke Akademie vanWetenschappen te Amsterdam. Section of Science. Proceedings 6: 809-831 (1904).

[10] Albert Einstein *"Zur Elektrodynamik bewegter Korper."* Annalen der Physik 18:639-641(1905).

[11] Minkowski

[12] GPS Physics Today

[13] L. C. Epstein, *Relativity Revisited (Insight Press San Fransisco1982)*. In this novel and lucid treatment of relativity it is mentioned that Galileo gave the name *imprago* to effects of momentum when he thought up *Mu*.

[14] Max Planck "*Zur Theorie des Gesetzes der Energieverteilung im Normal-spectrum.*" Deutsche Physikalische Gesellschaft. Verhandlungen 2: 237-245 (1900).

[15] Planck remorseful about E=h nu ?

[16] Albert Einstein "*Uber einen die Erzeugung und Verwandlung des Lichtes betreffenden heuristischen Gesichtspunkt.*" Annalen der Physik 17: 132-148 (1905).

[17] Louis de Broglie, Nature 112, 540 (1923); Annalen der Physik (10) 2 (1923).

[18] Bohr quantization

[19] W. Heisenberg

[20] P.A.M. Dirac bra-ket

[21] Einstein dice quote

[22] Evenson' experiments depend on high order beats to build its frequency counting chains.

[23] Hamilton Dirac Feynman phase

[24] E. Schrodinger, Annalen der Physik (4) 79 361 and 489 (1923); 80, 437 (1926); 81,109 (1926).

[25] Feynman action and L clock

[26] Einstein recoil Gedanken

[27] Dirac theory

[28] Carl Anderson

[29] Feynman's father

[30] (unpublished) A huge energy shift is used for sake of geometric clarity. Atoms usually lose less than one part in 10^9 or 10^{10} of their rest mass in optical emission. The atom in the example loses rest mass 1/2 M_2c^2 emitting a 3/8 M_2c^2 photon.

[31] R. P. Feynman, R. Leighton, and M. Sands, *The Feynman Lectures* (Addison Wesley 1964) The present development owes a lot to Feynman's treatment of wave frequency.

[32] Einstein elevator / equivalence principle

[33] E.F.Taylor and J.A. Wheeler, *Spacetime Physics* (W. H. Freeman San Francisco 1966) p. 18.

Acknowledgements
The author would like to thank Ronald F. Fox and Eric J. Heller for careful reading and suggestions regarding early versions of this work and Daniel Kennefick for information from the Einstein Centennial bibliography project.